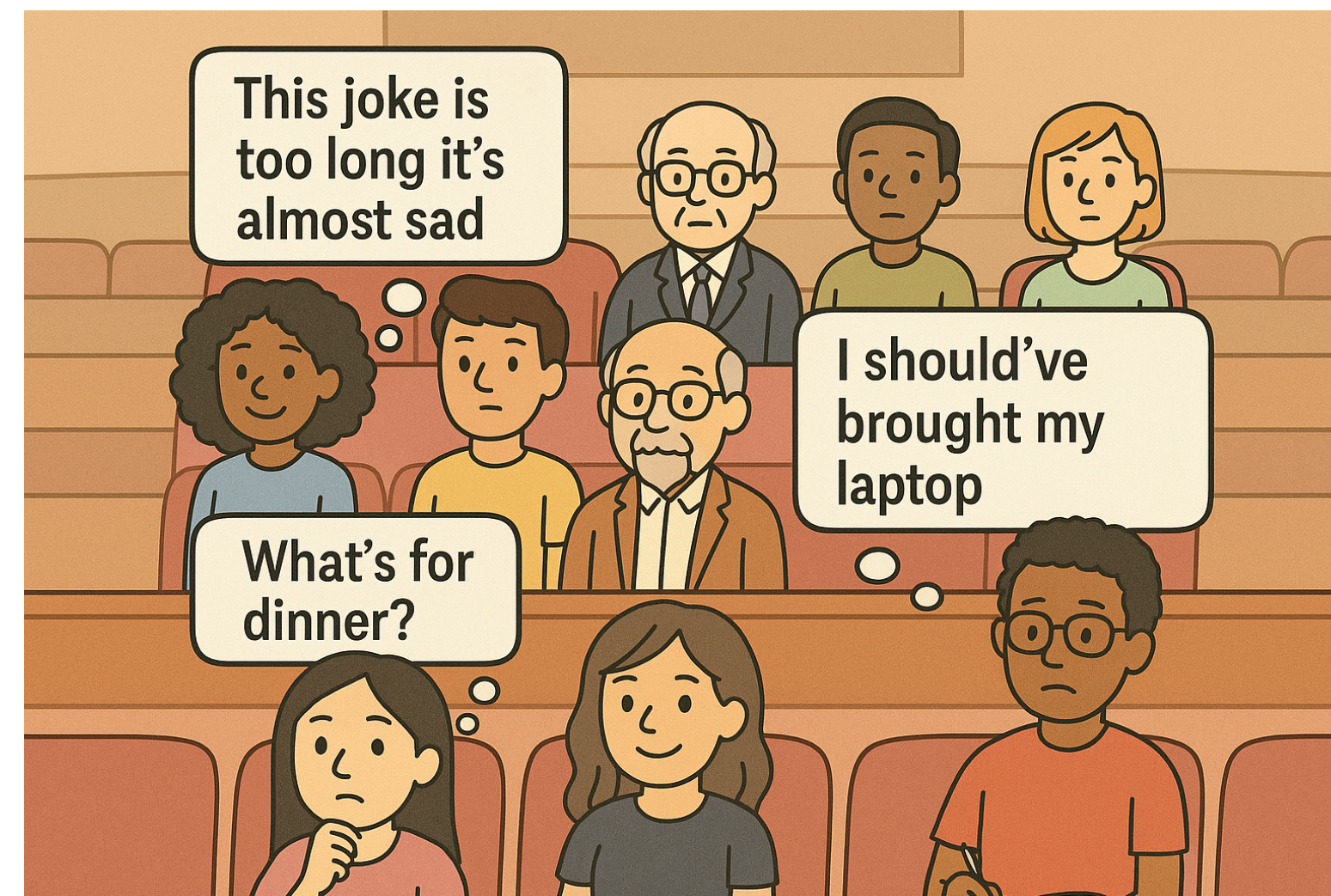
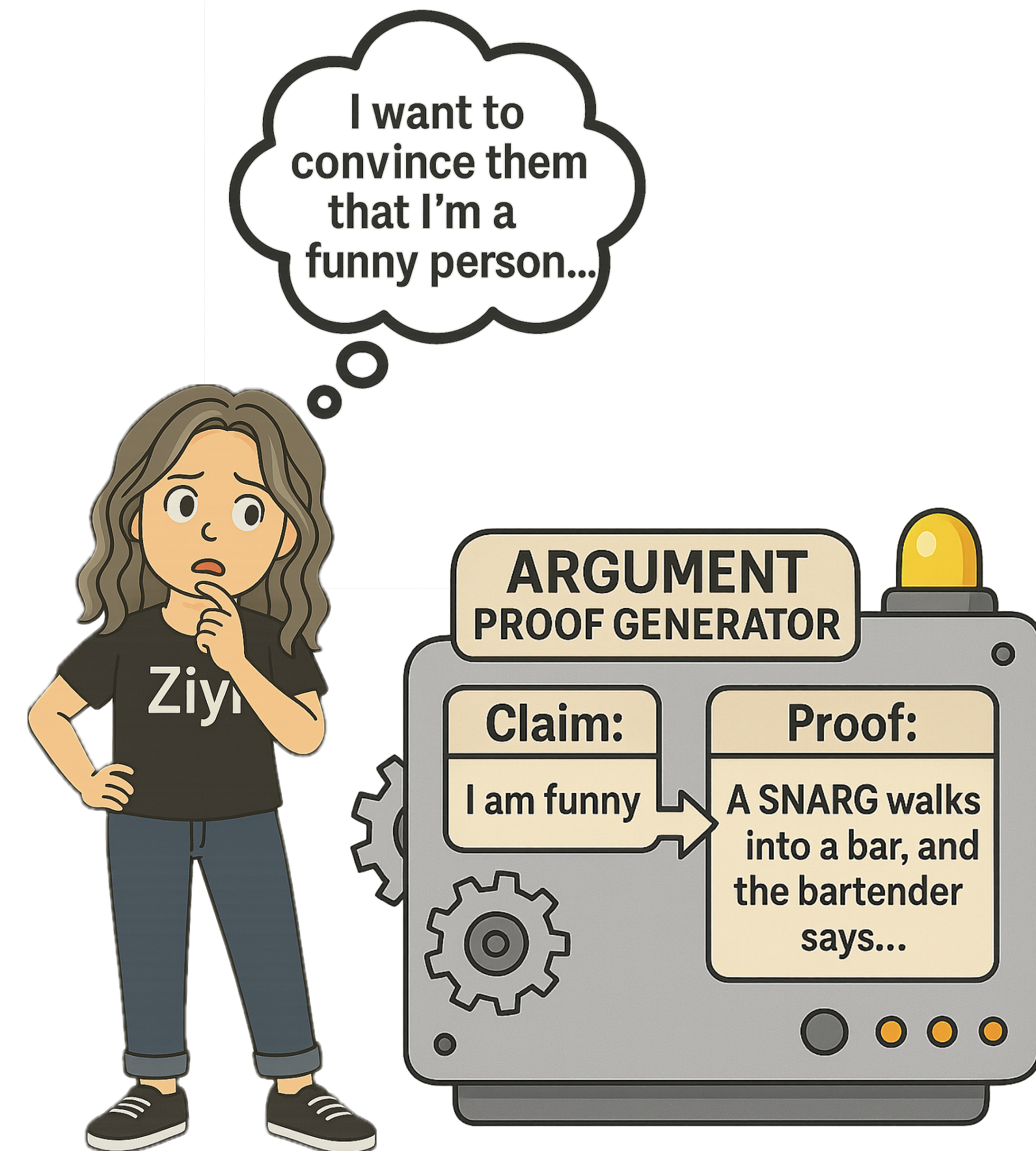


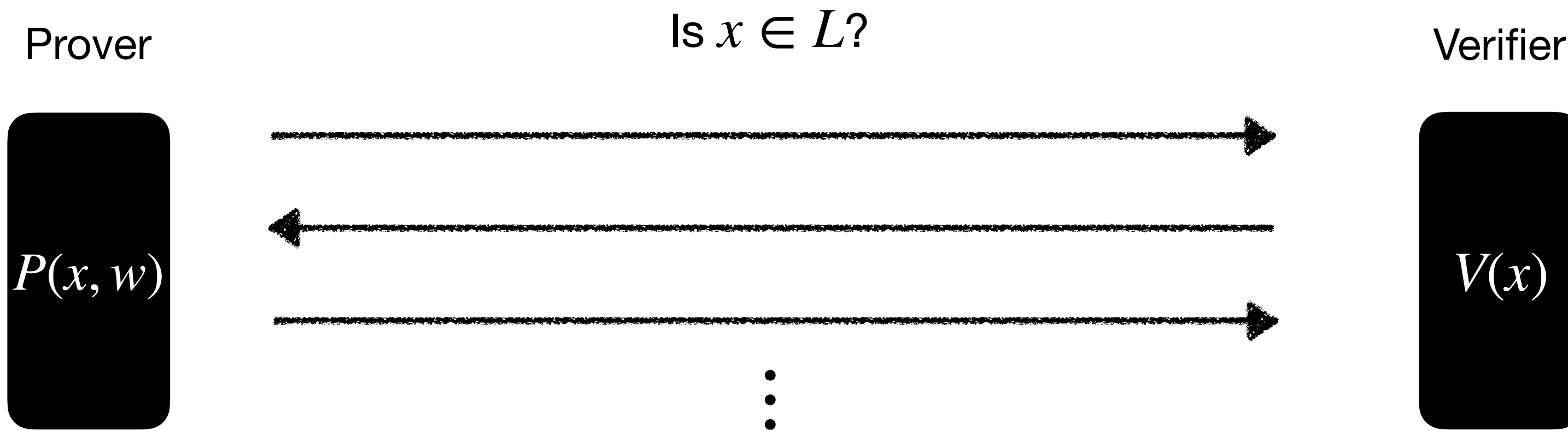
On the Security of Succinct Arguments from Probabilistic Proofs

Ziyi Guan



What are succinct arguments?

Interactive proofs



Completeness: $\forall x \in L, \Pr [\langle P(x, w), V(x) \rangle = 1] = 1$

Soundness: $\forall x \notin L$ and adversary \tilde{P} , $\Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon$

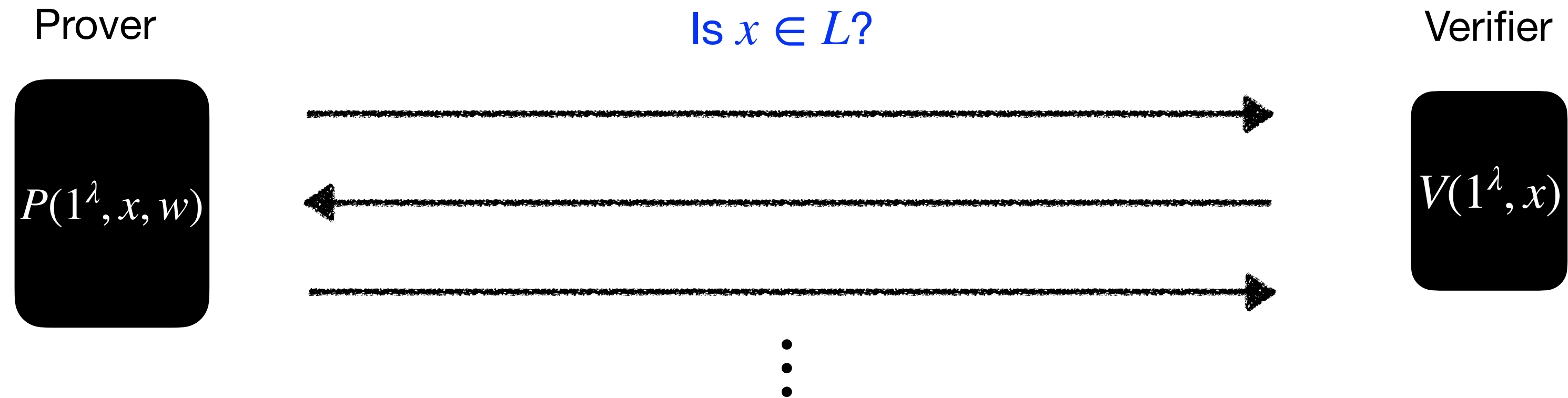
Target metric: **COMMUNICATION COMPLEXITY**

Limitation: NP-complete languages do not have IPs with **CC** $\ll |w|$

[GH97]: $\text{IP}[\text{CC}] \subseteq \text{BPTIME}[2^{\text{CC}}]$

Interactive arguments

Interactive proofs with **computational** soundness



Computational soundness: $\forall x \notin L$ and t_{ARG} -time adversary \tilde{P} , $\Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon_{\text{ARG}}(t_{\text{ARG}})$

AMAZING: \exists interactive arguments for NP with **CC** $\ll |w|$ (given basic cryptography)

Today's protagonist:
Succinct Interactive Arguments

Why study ^{$cc \ll |w|$} **succinct** ^{$time(V) \ll |w|$} interactive arguments?

They exist based on simple crypto assumptions...

... so they play a role in numerous cryptottheory results.

zero-knowledge with
non-black-box simulation

malicious MPC

...

They are a stepping stone for SNARGs, which have numerous real-world applications.

 **Succinct**

 **RISC
ZERO**

 **Aztec**

 **VALIDA**

Irreducible

 **STARKWARE**

 **polygon**

 **NEXUS**

 **Ligero**

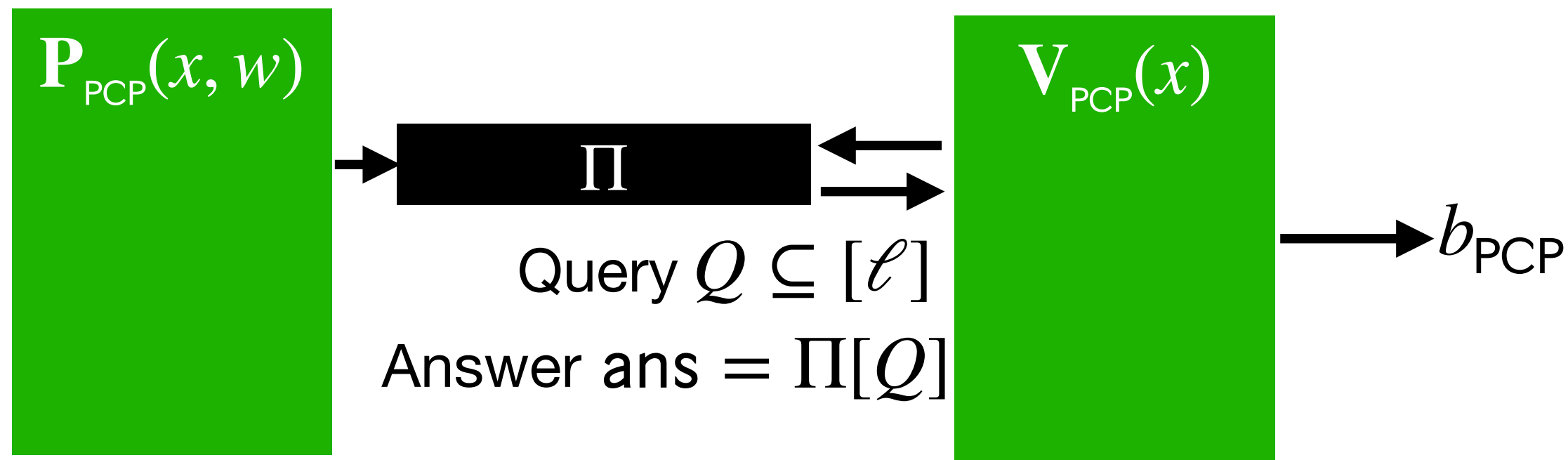
 **Matter Labs**

...

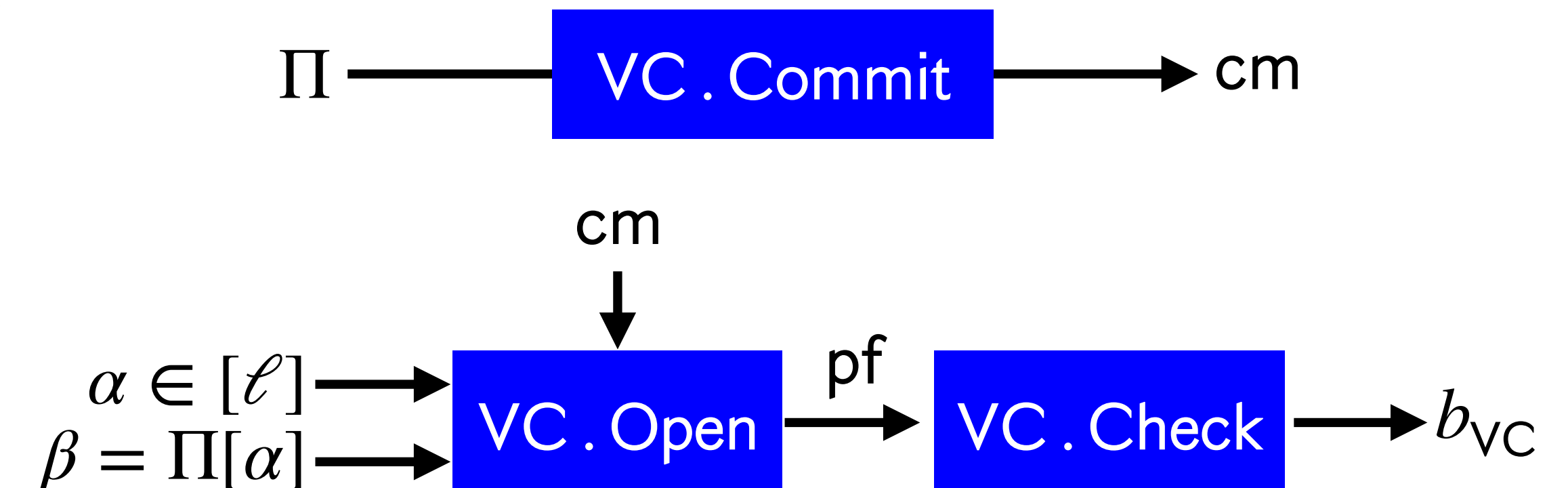
Kilian's protocol: The first and simplest succinct argument

How to construct succinct arguments?

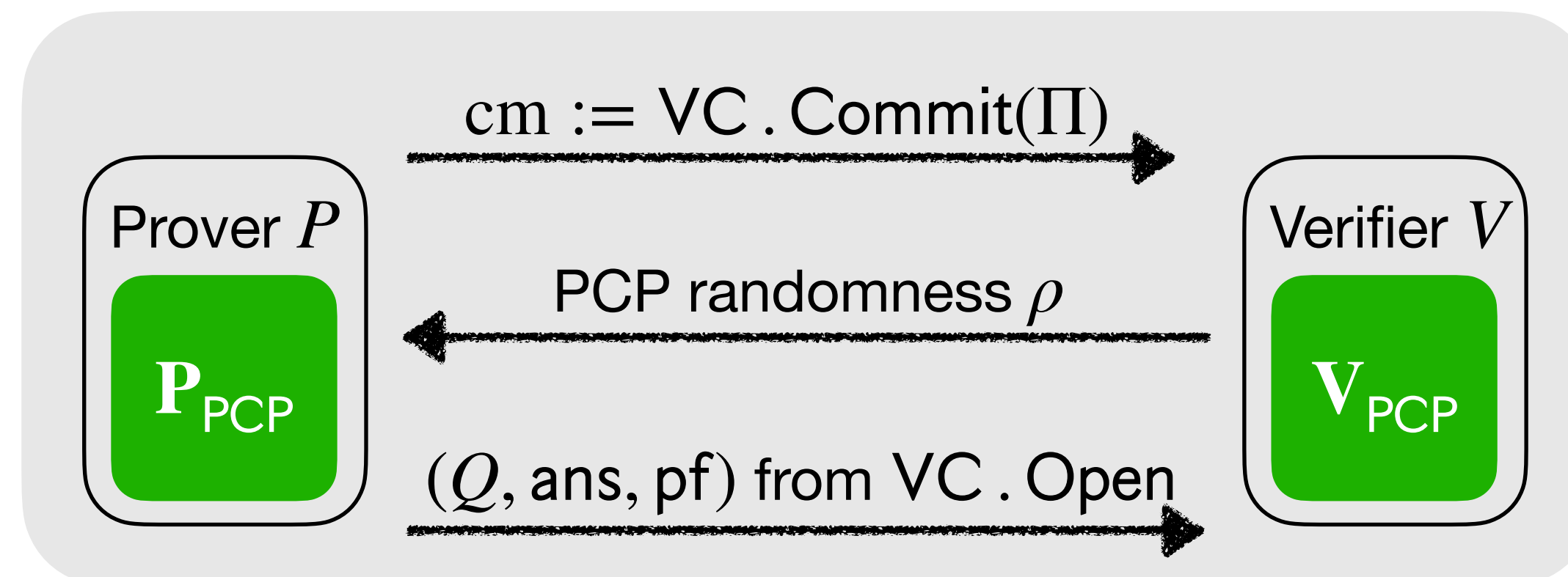
Building block #1: probabilistically checkable proof (PCP)



Building block #2: vector commitment scheme (VC)



Kilian's protocol



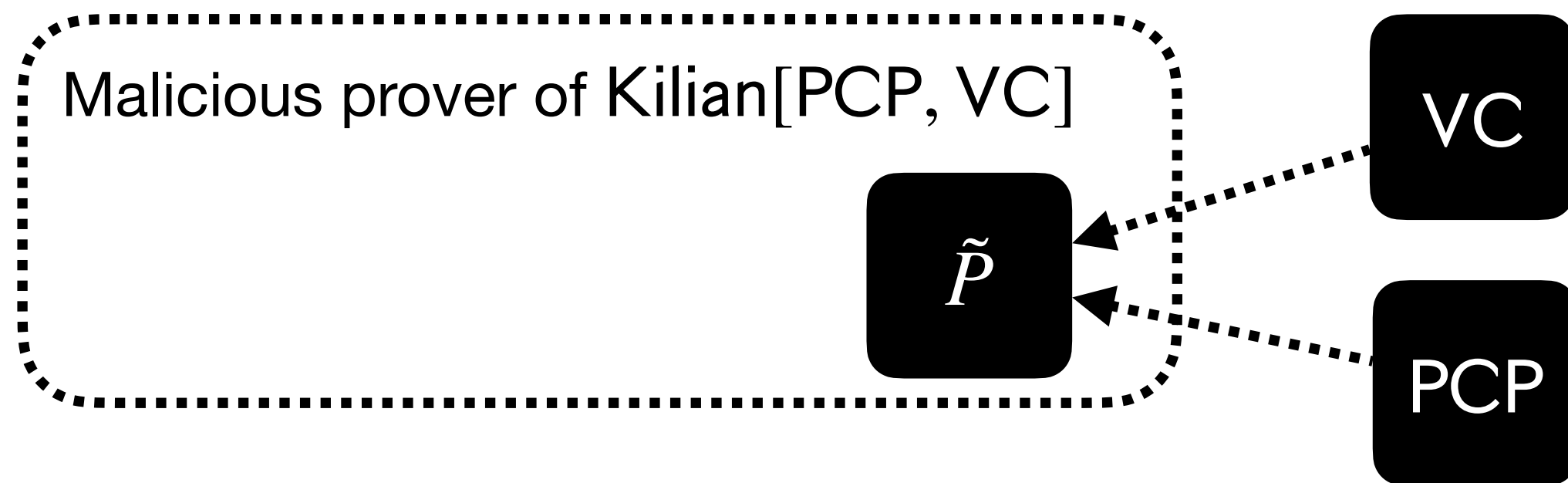
T -step computation:

- Prover time: $\text{poly}(T)$
- Verifier time: $\text{polylog}(T)$

(**cc**: $\text{polylog}(T)$)

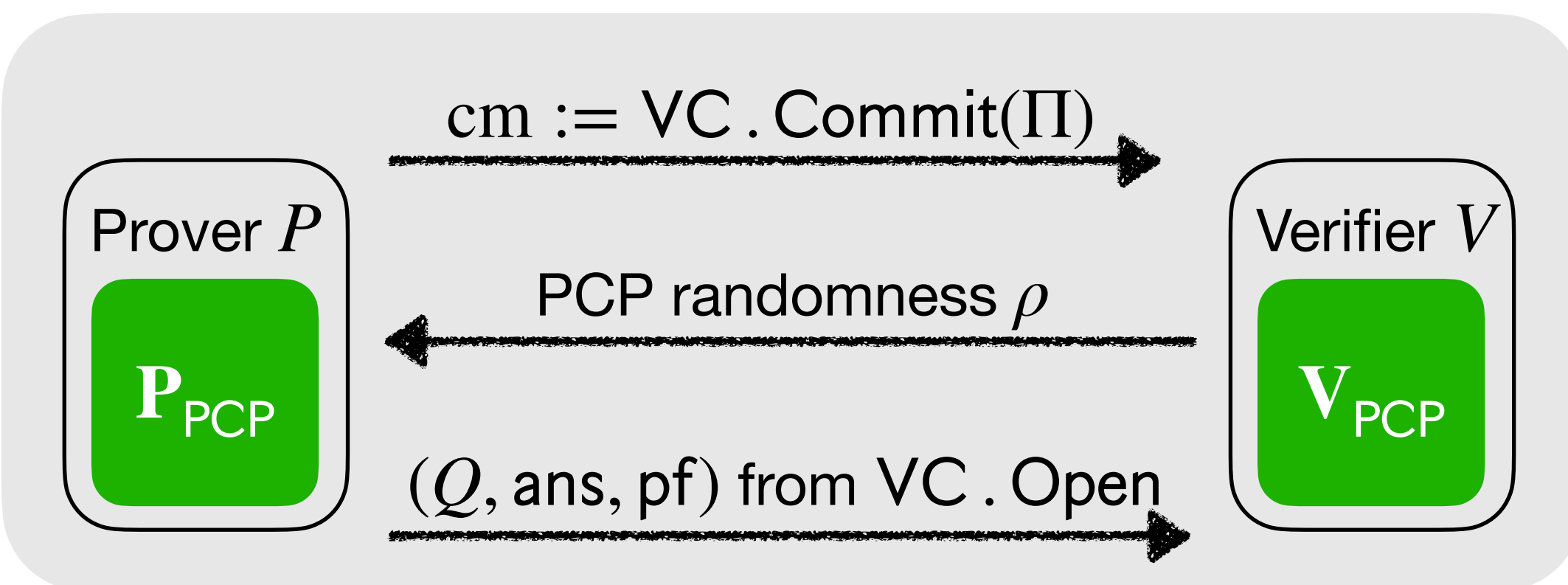
Simple (and only known) security analysis

Goal: relate the soundness error of Kilian[PCP, VC]
to the soundness error of PCP and the position binding error of VC.



Rewind \tilde{P} to get a malicious PCP string $\tilde{\Pi}$
 \Rightarrow (PCP soundness) upper bound the success probability of $\tilde{\Pi}$
 \Rightarrow (Position binding) $\tilde{\Pi}$ cannot be too different from \tilde{P}

Kilian's protocol



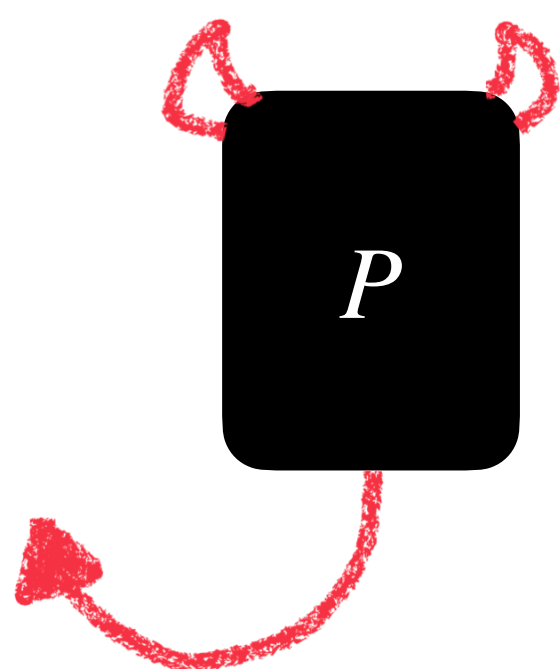
Position binding

$$\Pr \left[\begin{array}{l} (cm, Q, Q', ans, ans', pf, pf') \leftarrow Adv \\ \begin{array}{|c|c|c|c|c|c|c|} \hline \text{ } & \text{blue} & \text{ } & \text{blue} & \text{blue} & \text{blue} & \text{ } \\ \hline \end{array} \text{Check}(cm, Q, ans, pf) = 1 \\ \begin{array}{|c|c|c|c|c|c|c|} \hline \text{ } & \text{blue} & \text{ } & \text{red} & \text{ } & \text{ } & \text{blue} \\ \hline \end{array} \text{Check}(cm, Q', ans', pf') = 1 \end{array} \right] \leq \epsilon_{VC}$$

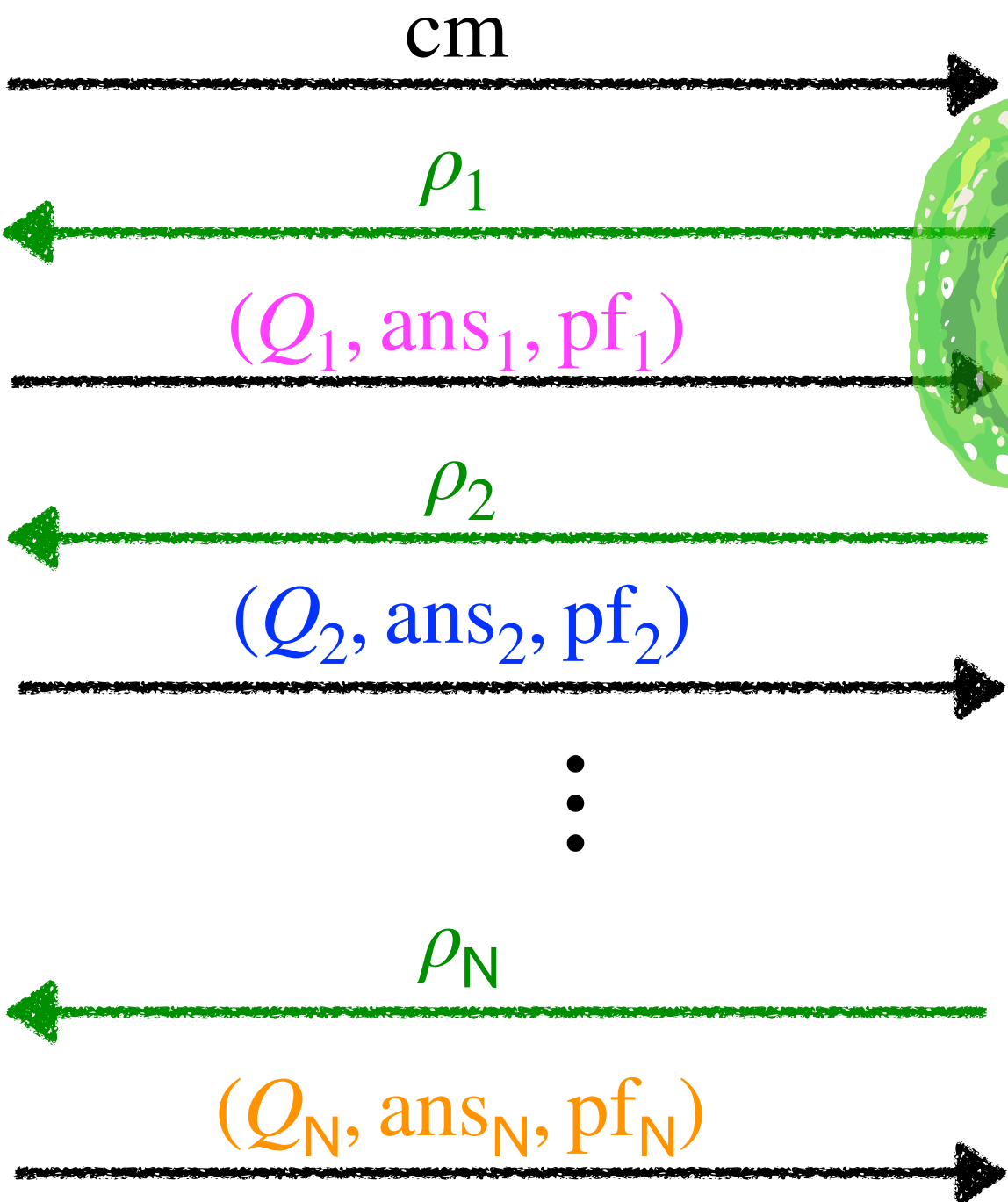
Security from rewinding

How to rewind?

Malicious Prover \tilde{P}

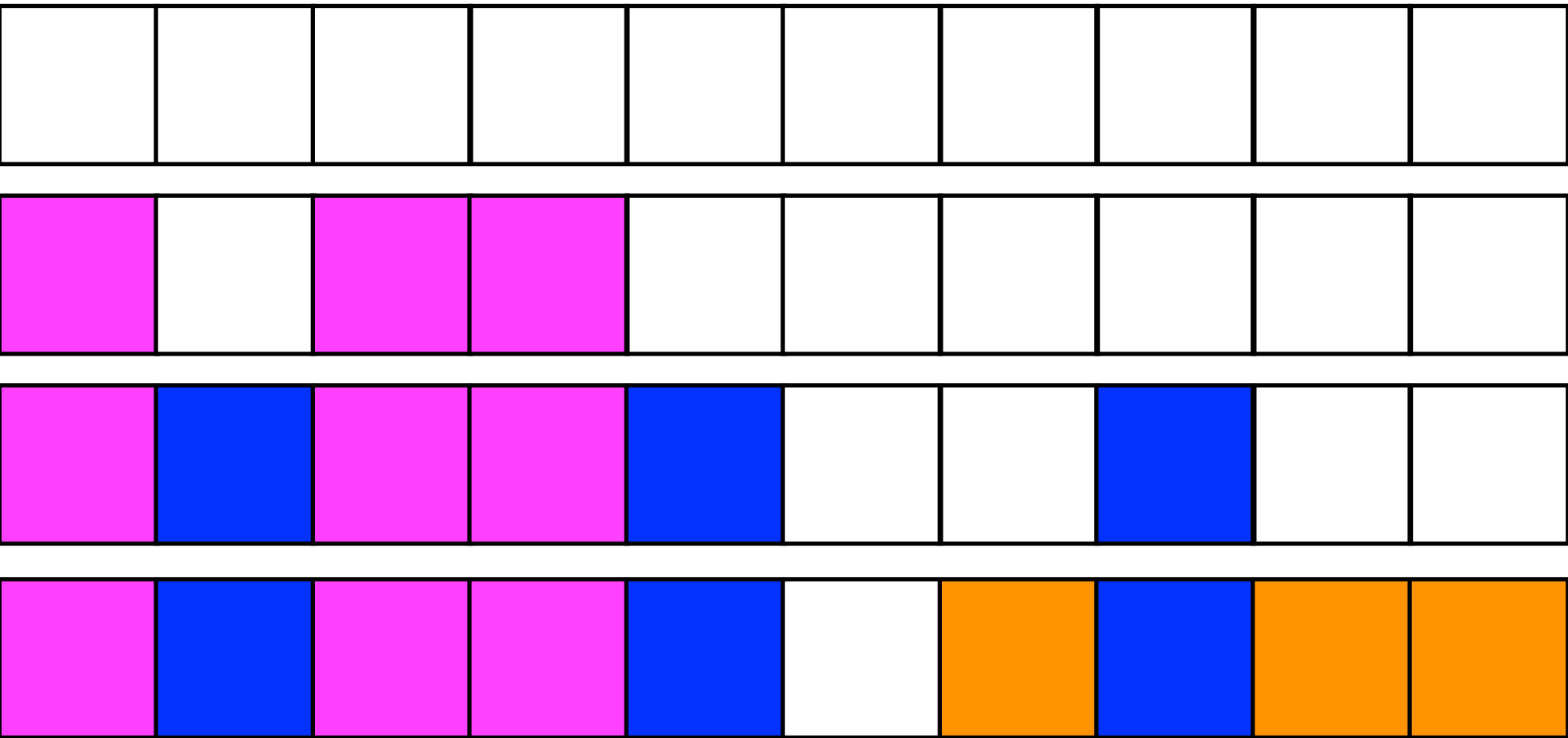


Reductor $\mathcal{R}^{\tilde{P}}(\text{cm}, \epsilon)$

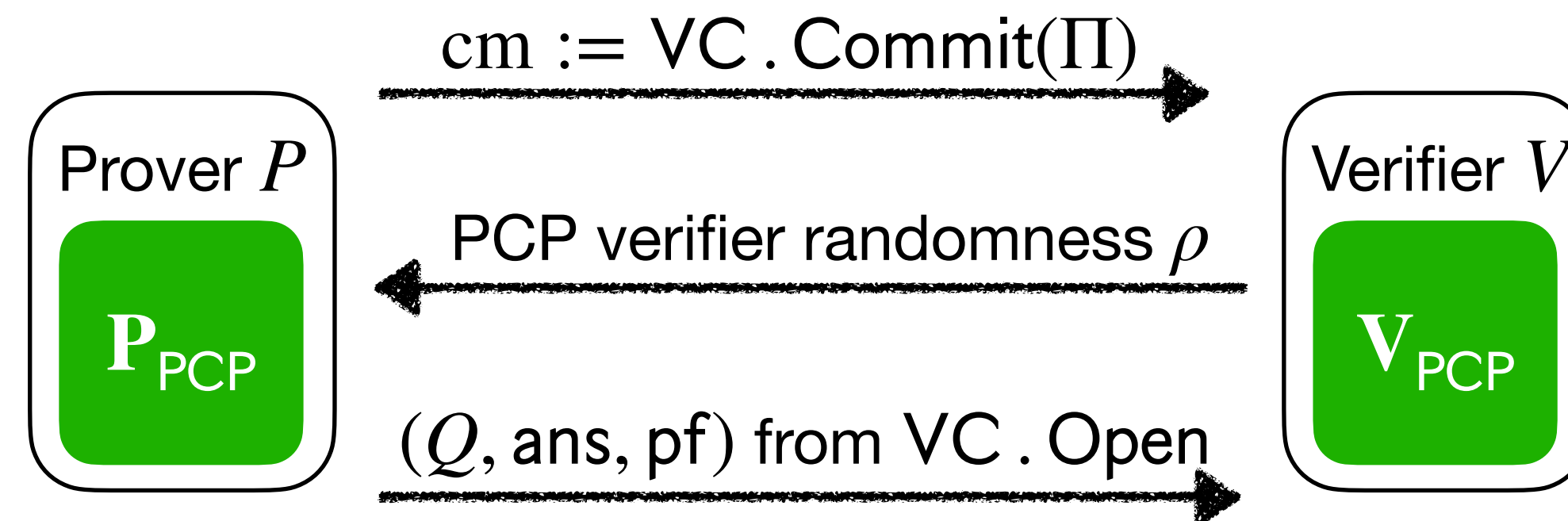


- Subtle design choices:
- Strict time vs. expected time
 - Sample with/without replacement
 - Stopping conditions
 - ...

Recover $\tilde{\Pi}$



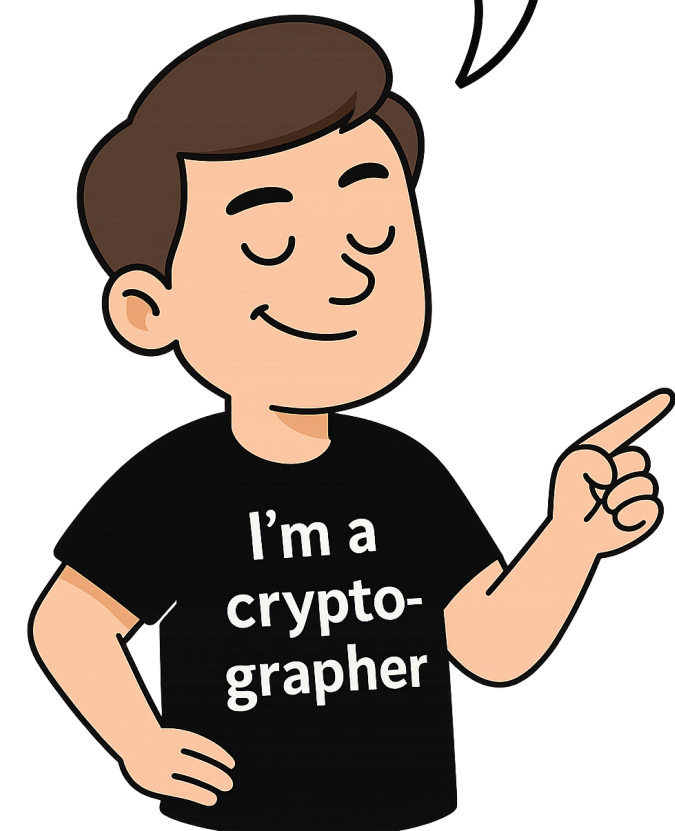
What is the security of Kilian's protocol?



Previously:

- [Kilian92] gives an **informal** analysis
- [BG08] $\epsilon_{ARG} \leq 8 \cdot \epsilon_{PCP} + \sqrt[3]{\epsilon_{VC}}$ and **assuming** PCP is **non-adaptive** & **reverse-samplable** (non-trivial restrictions)
- [CMSZ21] Kilian is secure when ϵ_{PCP} **negligible** (in a paper about post-quantum security)

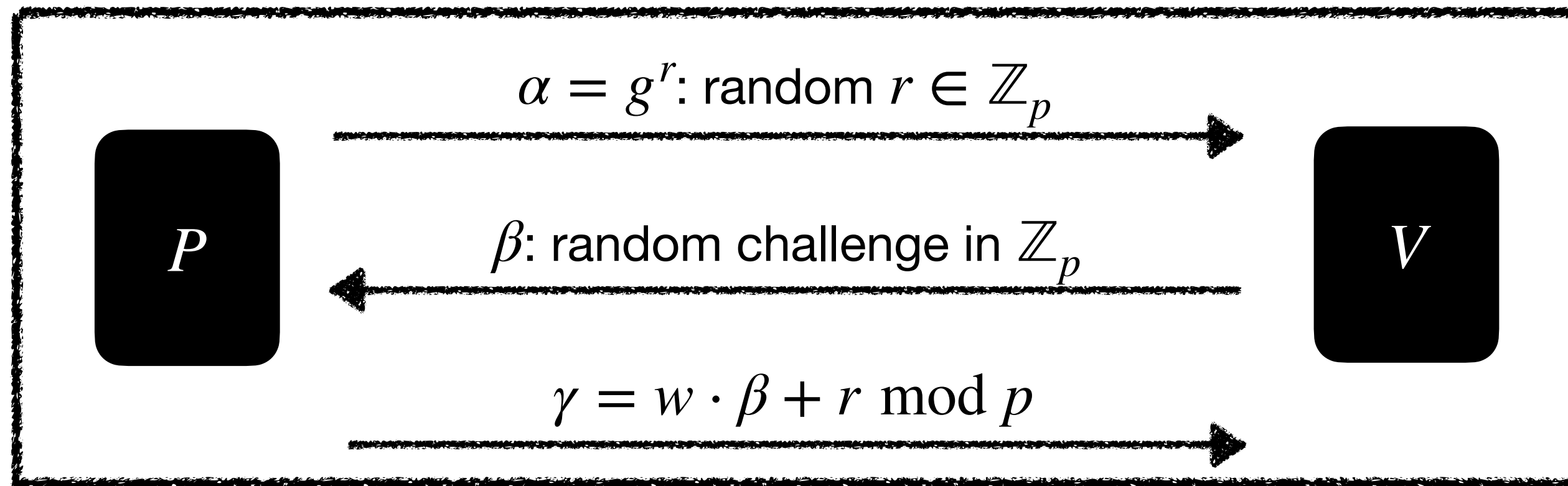
Of course,
this is trivial



We expect that $\epsilon_{ARG} \leq \epsilon_{PCP} + \epsilon_{VC} \dots$ right?

Surprise! A limitation:

$$\epsilon_{\text{ARG}} \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}} \implies \text{breakthrough on Schnorr}$$



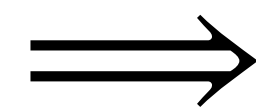
Lots of work on Schnorr security
[Sho97,PS00,BP02,FPS20,BD20,RS21,SSY23] ...
and yet there are still open questions on its optimal security!

Theorem. \exists PCP and VC s.t.

$$\epsilon_{\text{Schnorr}}(t) \leq \epsilon_{\text{ARG}}(t).$$

Similar bound holds for
expected-time adversary

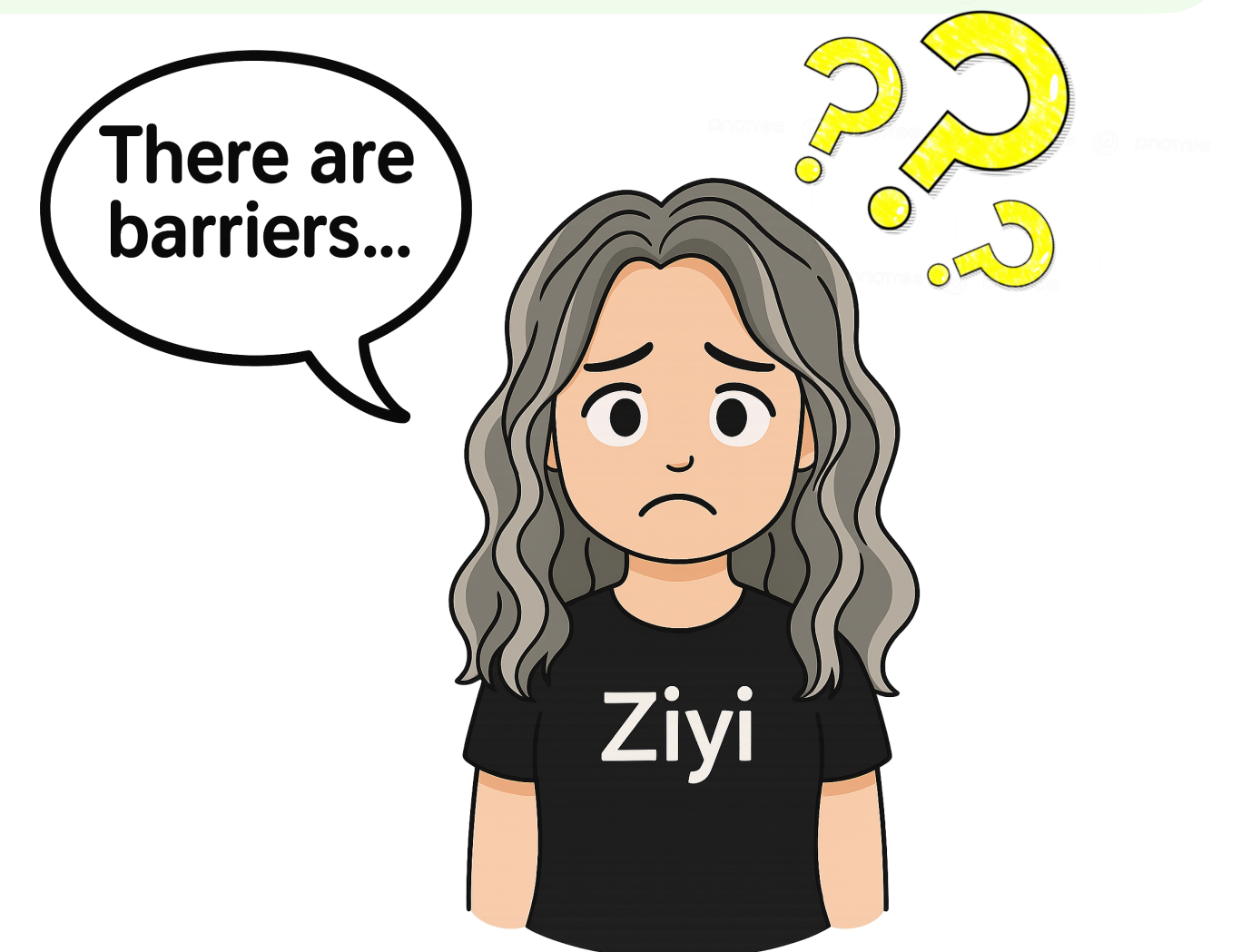
Suppose
 $\epsilon_{\text{ARG}} \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}$



$$\epsilon_{\text{Schnorr}}(t_{\text{Schnorr}}) \leq \epsilon_{\text{DLOG}}(O(t_{\text{Schnorr}}))$$

Best analysis of Schnorr [PS00]: $\epsilon_{\text{Schnorr}}(t_{\text{Schnorr}}) \leq \sqrt{\epsilon_{\text{DLOG}}(O(t_{\text{Schnorr}}))}$

... so the folklore is beyond current rewinding techniques



Improved security for Kilian



Theorem. $\forall \epsilon > 0$,

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot l \cdot 1/\epsilon).$$

Why $l \cdot 1/\epsilon$ overhead?

- l locations in Π
- \implies Rewind at least l times (e.g. maybe all PCP queries but 1 are fixed)
- Some rewinds yield garbage:
 - The locations were already found
 - VC check fails
- \implies Need $1/\epsilon$ times for each location as buffer

λ : security parameter

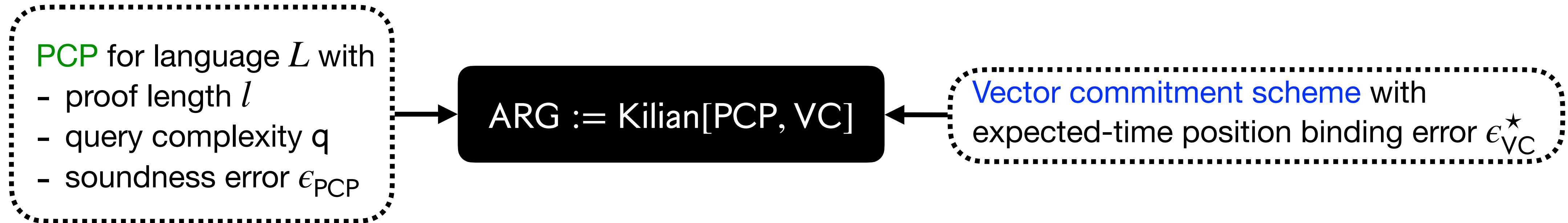
Suppose $\epsilon_{\text{VC}}(t) \leq O(t^2/2^\lambda)$ (e.g. an ideal Merkle tree)

By **Theorem**:

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{PCP}} + l^{2/3} \cdot O\left(\sqrt[3]{t_{\text{ARG}}^2/2^\lambda}\right)$$

That is, $\epsilon_{\text{ARG}} \leq \epsilon_{\text{PCP}} + \sqrt[3]{\epsilon_{\text{VC}}}$

Alternative route: expected-time regime



Theorem. $\forall \epsilon > 0$,

$$\epsilon_{\text{ARG}}^*(t_{\text{ARG}}^*) \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}^*(t_{\text{VC}}^*) + \epsilon, \text{ where } t_{\text{VC}}^* = O(t_{\text{ARG}}^* \cdot \log(q/\epsilon)).$$

λ : security parameter

Set $\epsilon_{\text{VC}}^*(t^*) \leq O\left(\sqrt{(t^*)^2/2^\lambda}\right)$ (e.g. an ideal Merkle tree)



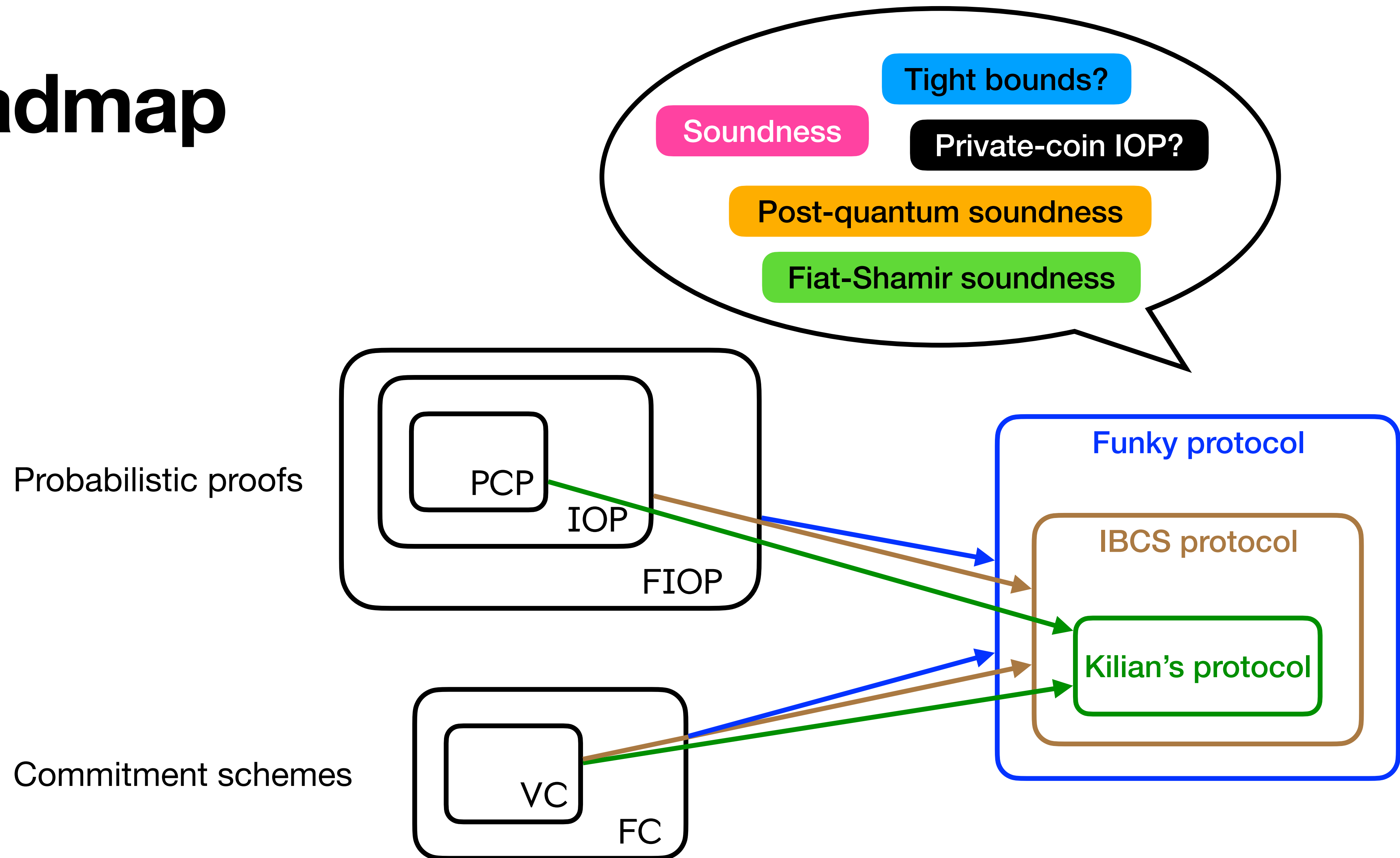
Theorem
 \Rightarrow

$$\begin{aligned} \epsilon_{\text{ARG}}^*(t_{\text{ARG}}^*) &\leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}^*(t_{\text{ARG}}^* \cdot \log(q/\epsilon)) + \epsilon \\ &\leq \epsilon_{\text{PCP}} + \text{polylog}\left(q \cdot \sqrt{(t_{\text{ARG}}^*)^2/2^\lambda}\right) \cdot O\left(\sqrt{(t_{\text{ARG}}^*)^2/2^\lambda}\right) \end{aligned}$$

small factor

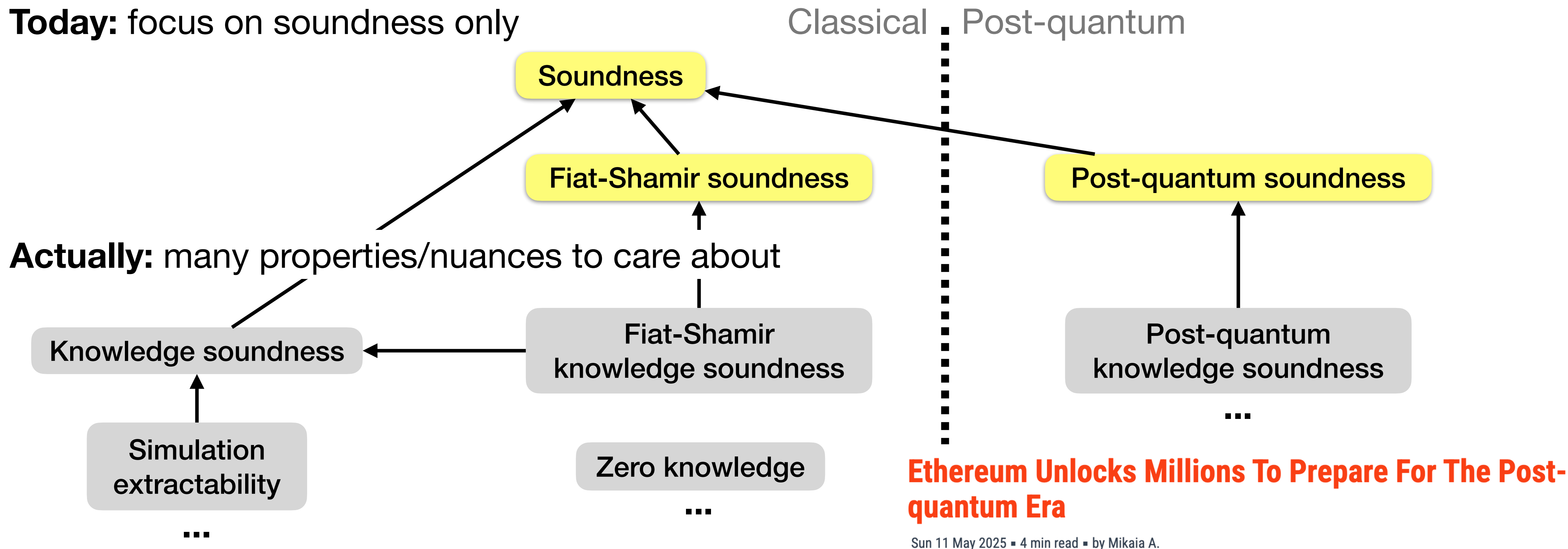
We achieved $\epsilon_{\text{ARG}}^* \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}^*$!

Roadmap



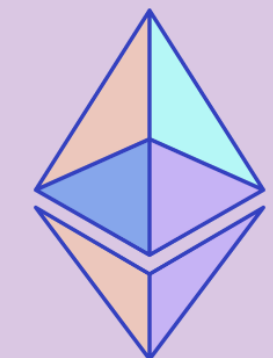
On security notions of arguments

Today: focus on soundness only



Strict-time adversary
vs.
Expected-time adversary

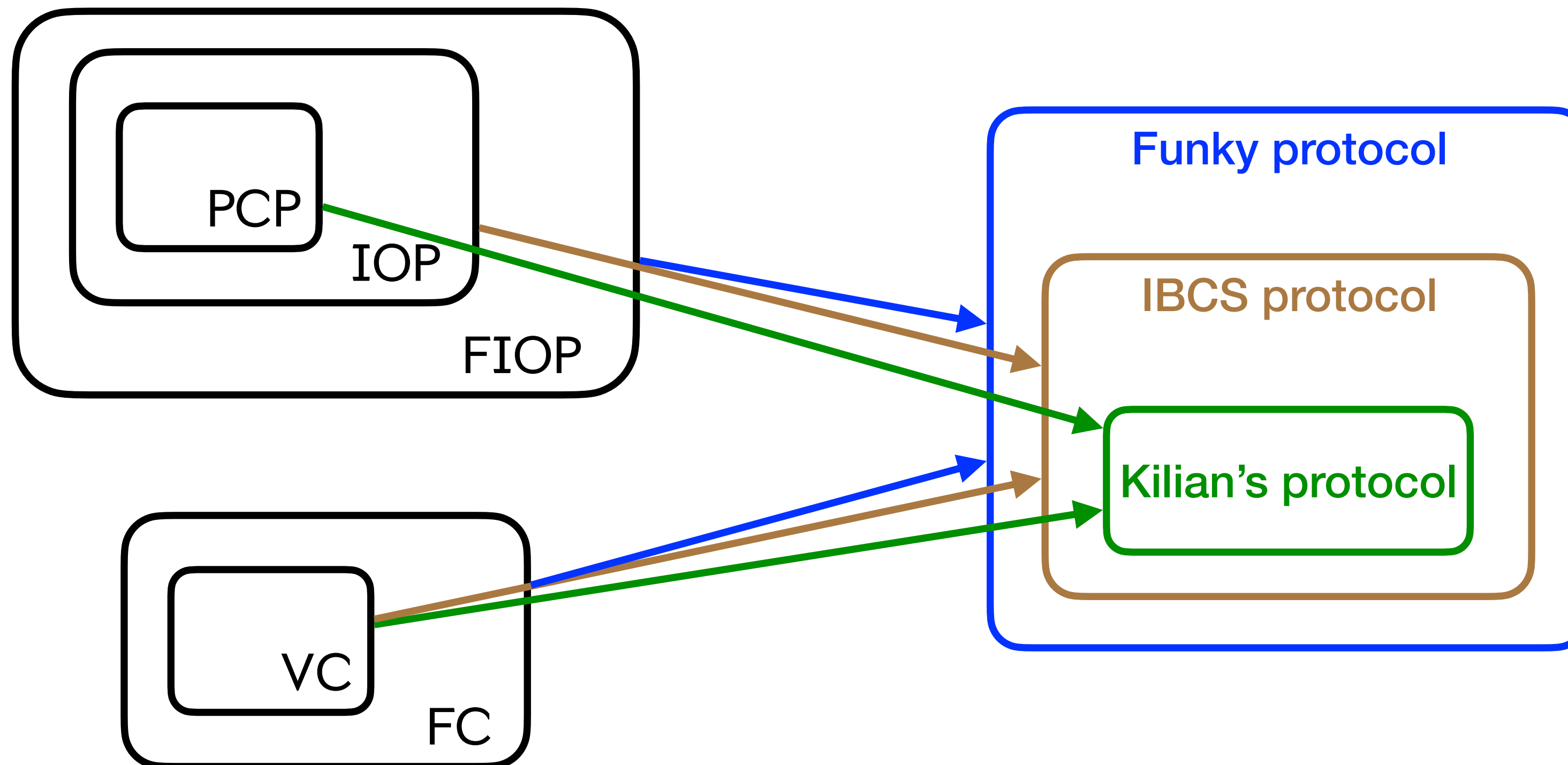
e.g., [BL02] zero-knowledge protocols do not have strict poly-time (black-box) extractor



zkEVM Formal Verification Project

A project by the Ethereum Foundation to accelerate the application of formal verification methods to zkEVMs

IBCS protocol: Using IOPs instead of PCPs

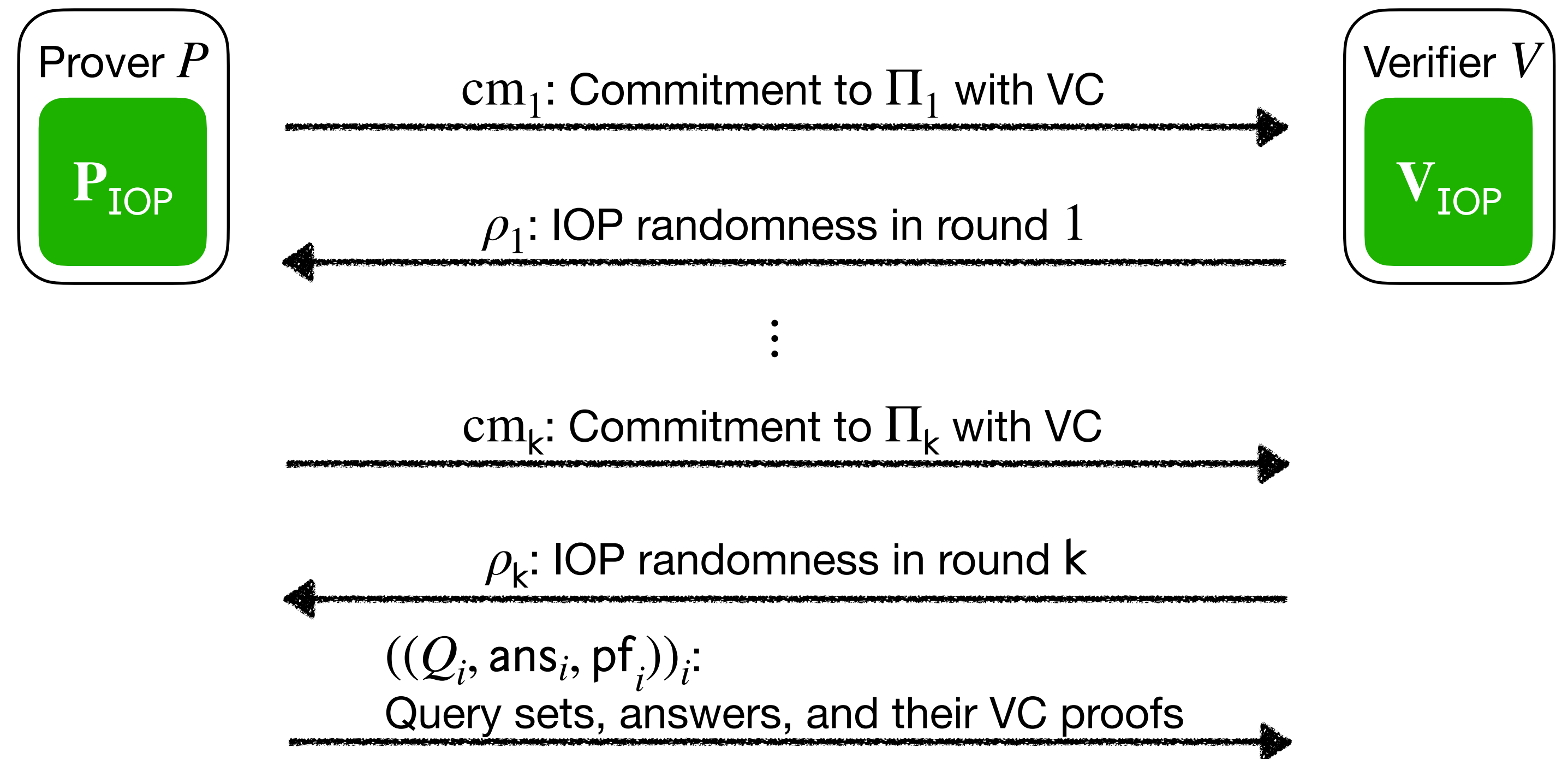
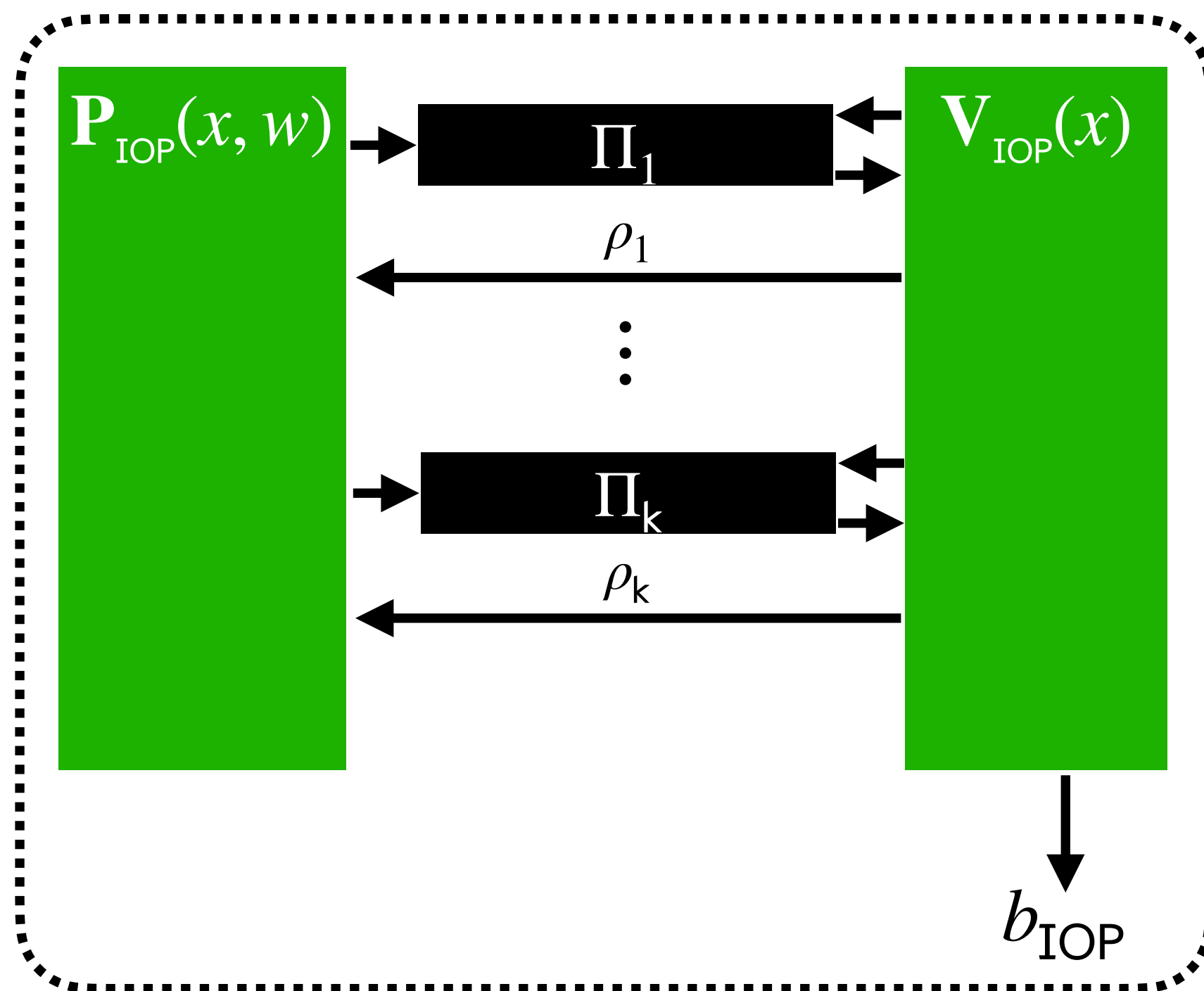


IBCS protocol

Existing PCPs are not concretely efficient: prover time too big

People use IOPs

Public-coin **interactive oracle proof** (IOP)



Security of IBCS protocol



The ideal bound ~~$\epsilon_{\text{ARG}} \leq \epsilon_{\text{IOP}} + \epsilon_{\text{VC}}$~~ is not possible... What can we get?

Theorem. $\forall \epsilon > 0$,

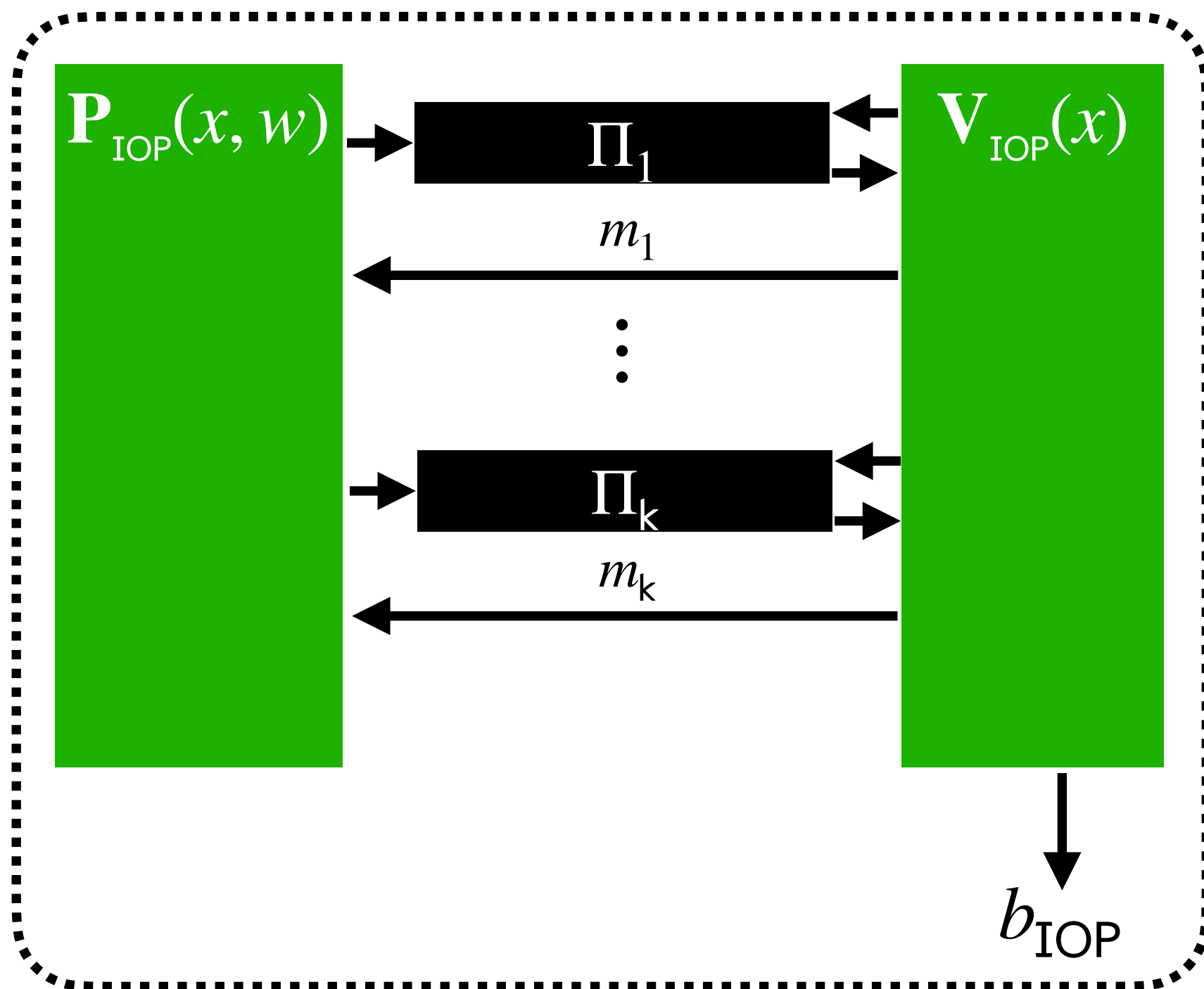
$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + \textcolor{blue}{k} \cdot \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot l/\epsilon).$$

Recall, for Kilian's protocol: $\forall \epsilon > 0$,

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{PCP}} + \textcolor{blue}{1} \cdot \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot l/\epsilon).$$

Why do we need public-coin IOPs?

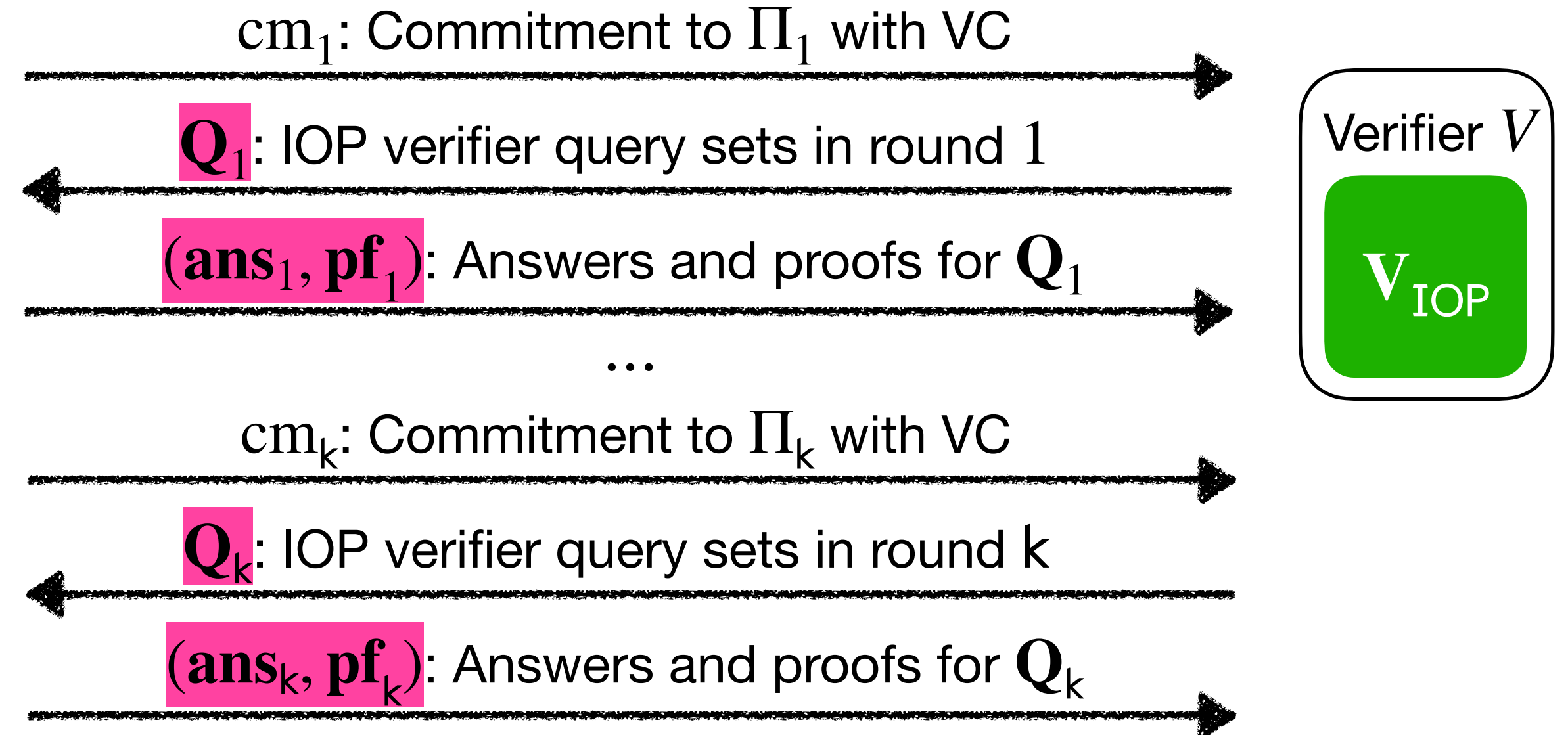
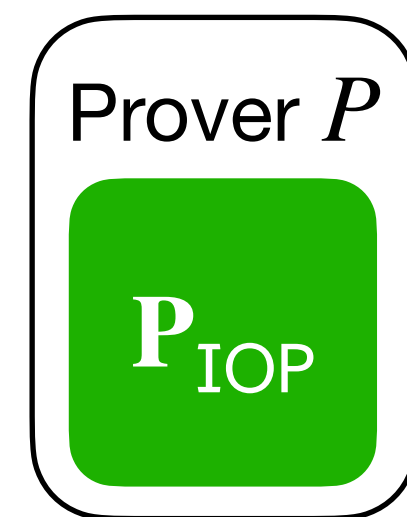
Private-coin **interactive oracle proof** (IOP)



How about public-query IOPs?

Queries can be learned by the prover (in "real-time")

Q_i contains verifier's queries to Π_1, \dots, Π_i

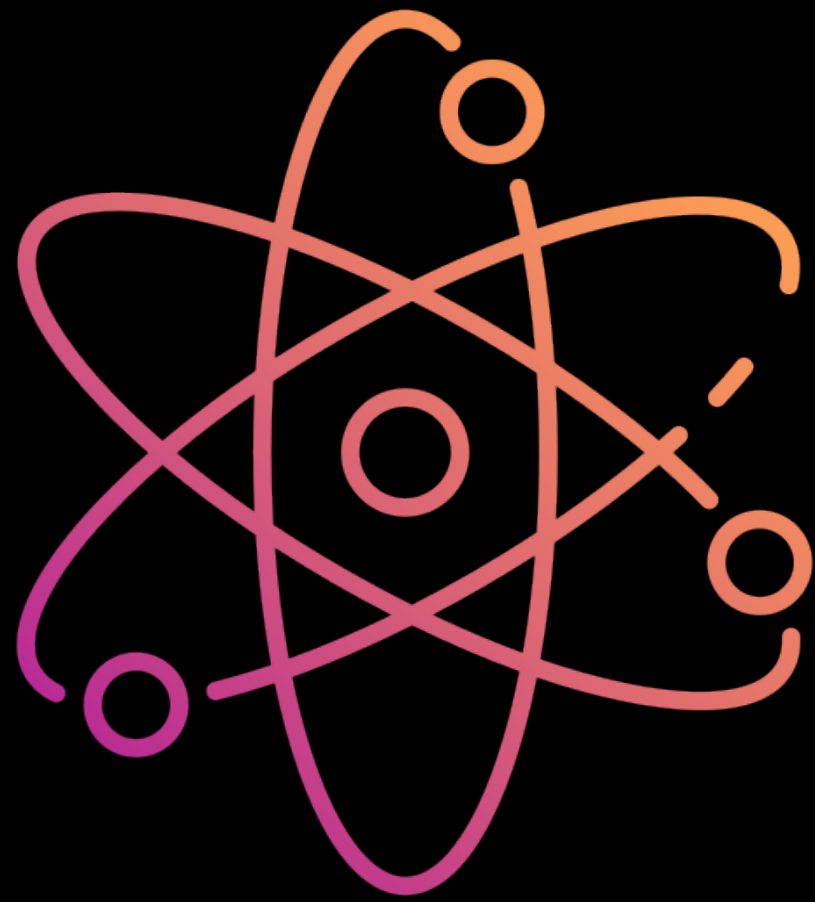


Not secure!
e.g. IOP verifier accepts if IOP prover guesses all its queries

Clearly, the IBCS protocol is secure whenever the underlying IOP is public-query... right?

Lemma: secure if IOP has an "efficient random continuation sampler"

Open question: can we prove security for ALL public-query IOPs?
(Or maybe there is a black-box barrier?)

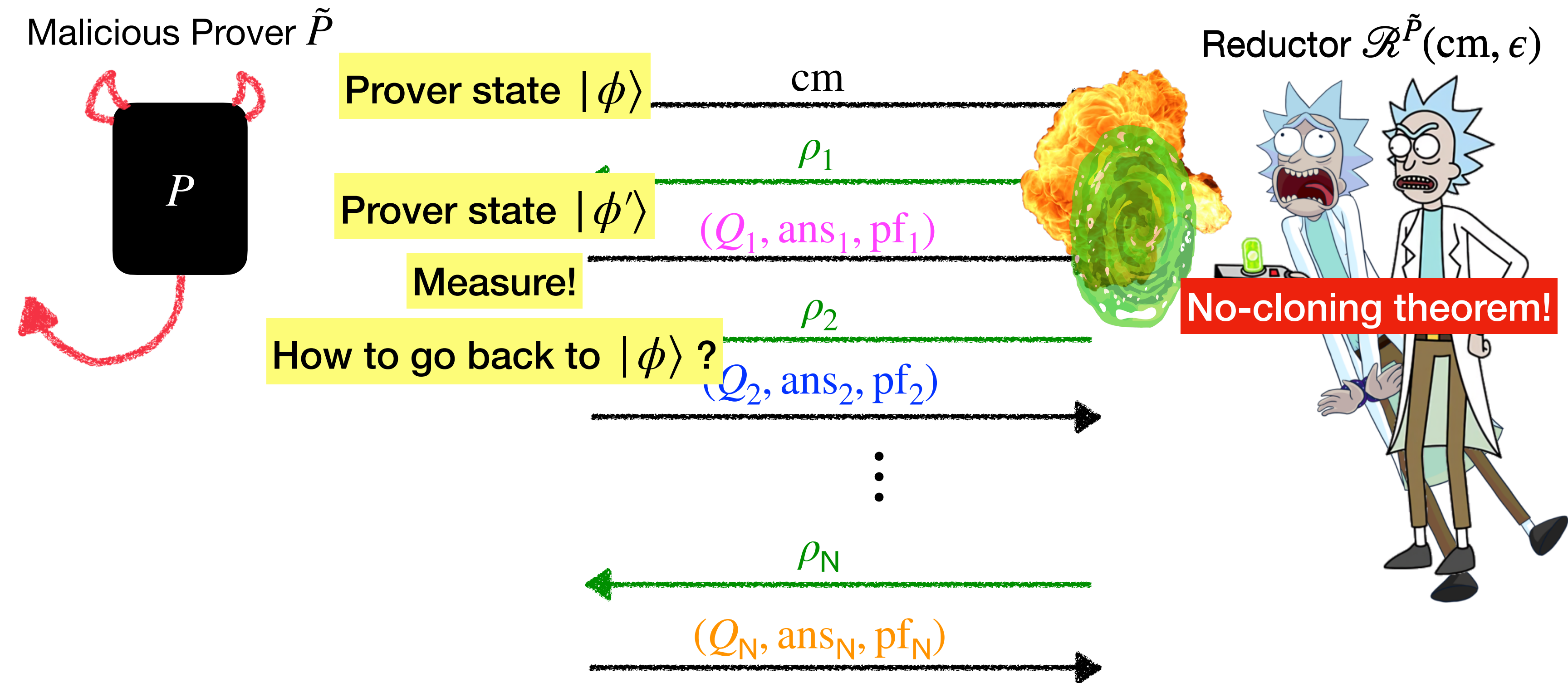


Interlude: **post-quantum** security

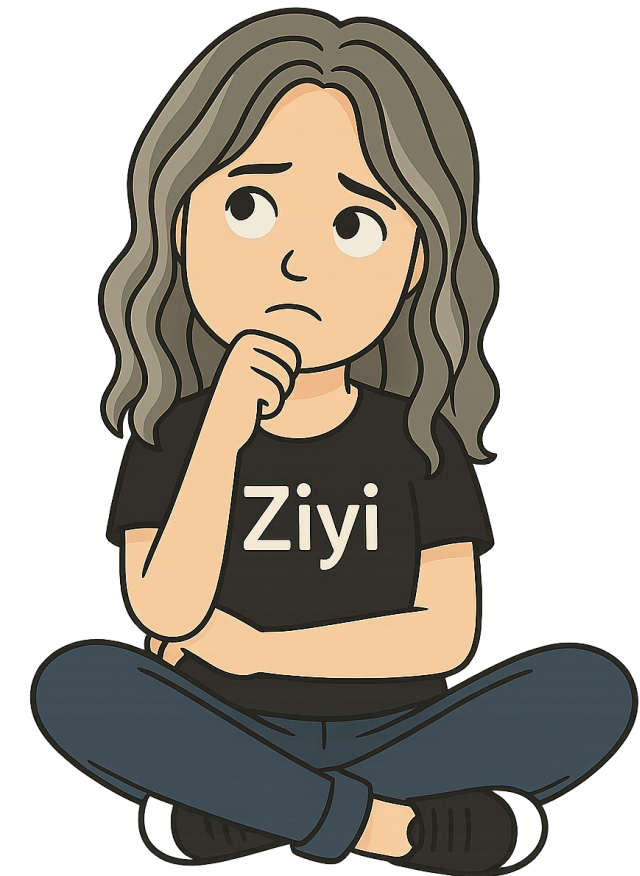
Post-quantum soundness: same as classical soundness but adversary is quantum

$$\forall t_{\text{ARG}}\text{-time } \mathbf{QUANTUM} \text{ adversary } \tilde{P}, \Pr [\langle \tilde{P}, V \rangle = 1] \leq \epsilon_{\text{ARG}}(t_{\text{ARG}})$$

On quantum rewinding



But rewinding is everywhere in crypto, how did people prove anything without it?



For many years: can rewind $O(1)$ times [Wat06, Unr12, Unr16b]

Problem: Kilian's protocol needs many rewindings

Recent new tools for quantum rewinding [CMSZ21]:
 “repair” the state instead of “rewind”
 \Rightarrow post-quantum security of Kilian's protocol

- Quantum rewinding toolset is cumbersome
- Only other paper studying many-round interactive arguments [LMS22] had to white-box adapt the tools in [CMSZ21]... (work for log rounds)

Adapting for IBCS protocol runs into challenges

Post-quantum security of IBCS protocol



Technical contribution: We build on [CMSZ21] and more...

Theorem. $\forall \epsilon > 0$,

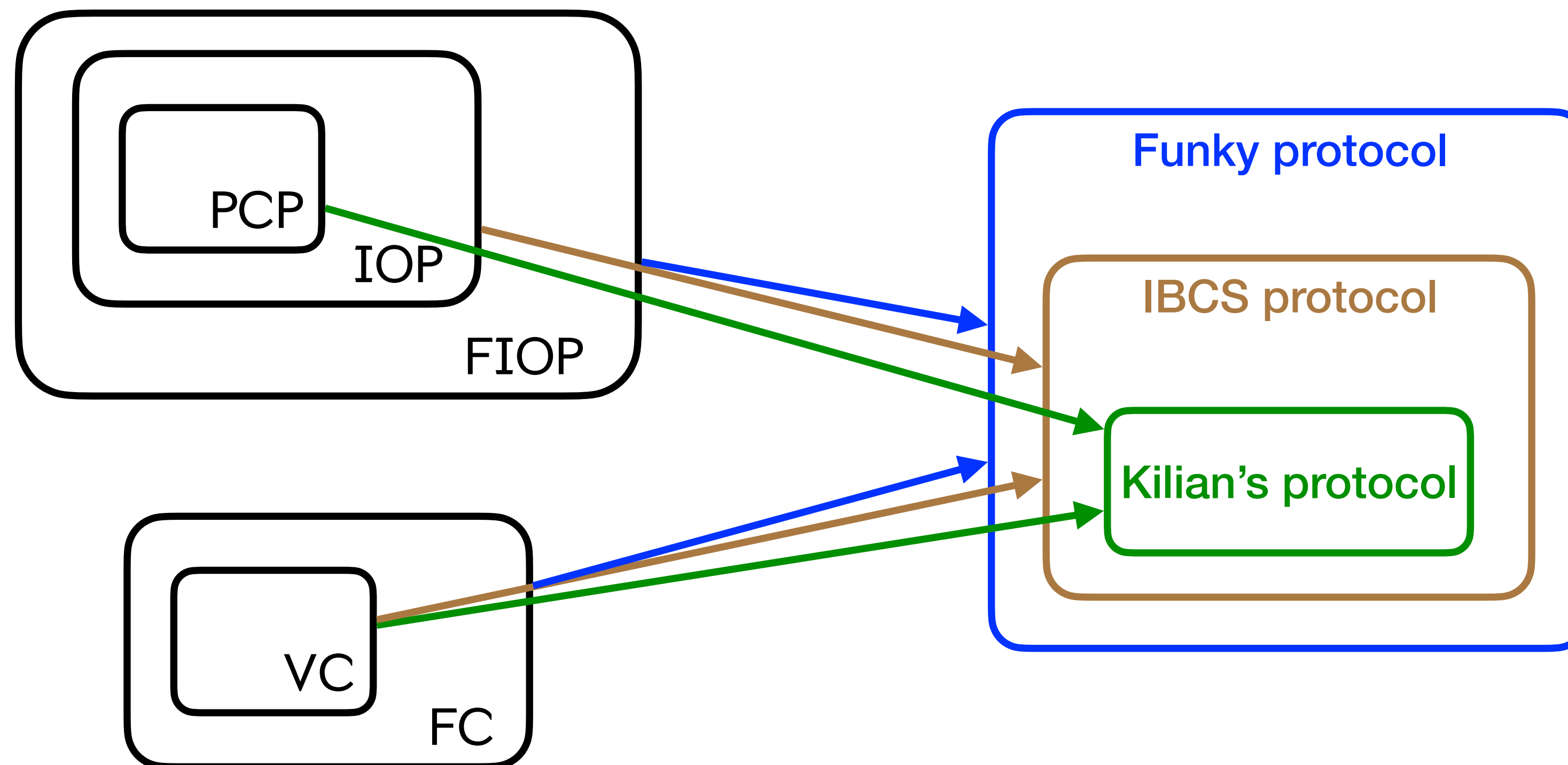
$$\epsilon_{\text{ARG}}^{\text{PQ}}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + k \cdot l \cdot \epsilon_{\text{VCCollapse}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = \text{poly}(t_{\text{ARG}} \cdot l/\epsilon).$$

Extra l factor: cost of quantum rewinding

$$\text{IBCS soundness: } \epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + k \cdot \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot l/\epsilon).$$

Corollary: post-quantum secure succinct arguments in the standard model (no oracles), with the best asymptotic complexity known.

Funky protocol: Construction from all probabilistic proofs

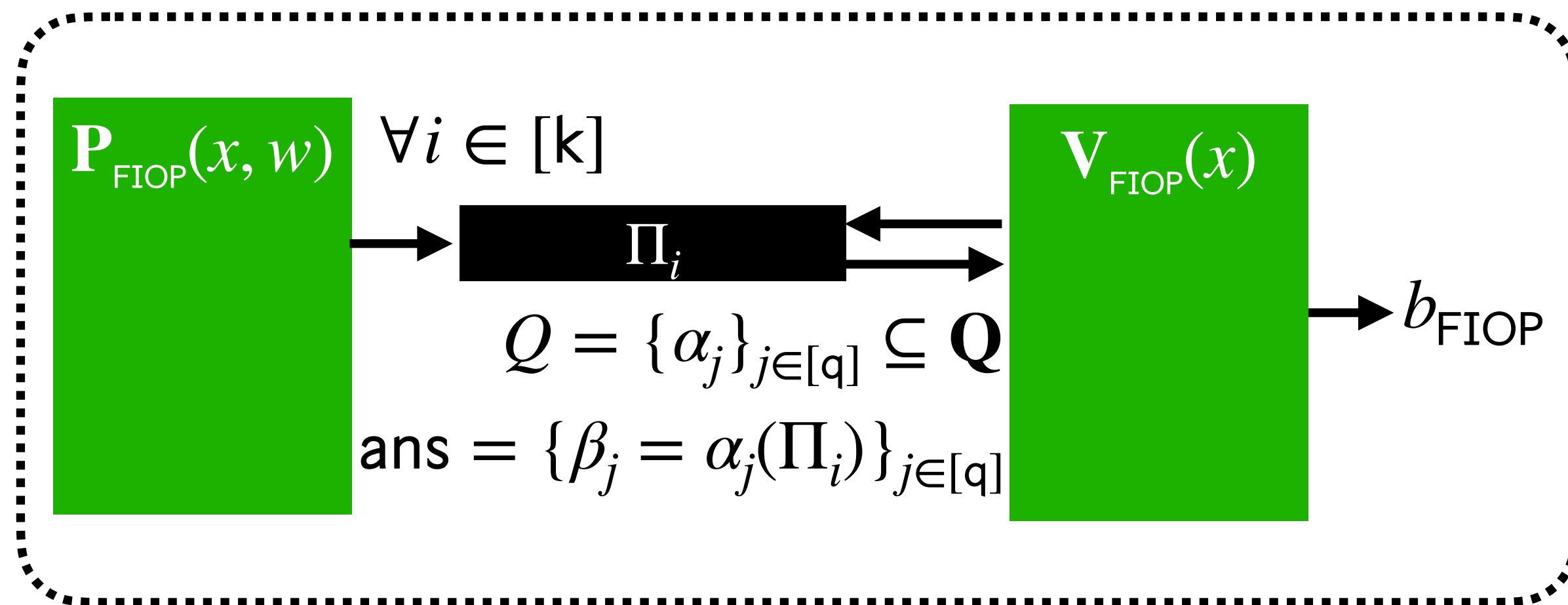


Building blocks

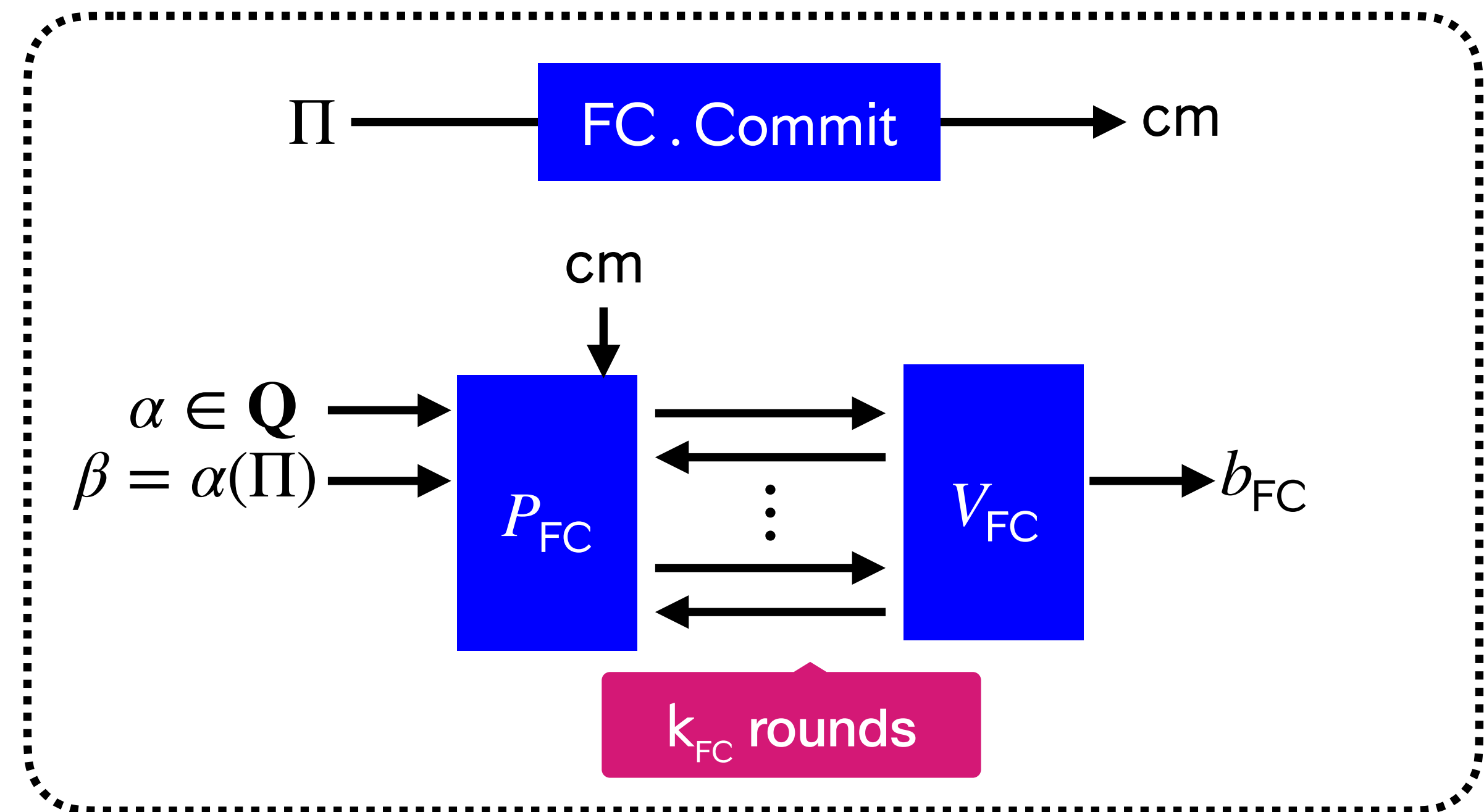
Building block #1: query class \mathbf{Q}

- $\mathbf{Q} \subseteq \{\alpha: \Sigma^\ell \rightarrow \mathbb{D}\}$

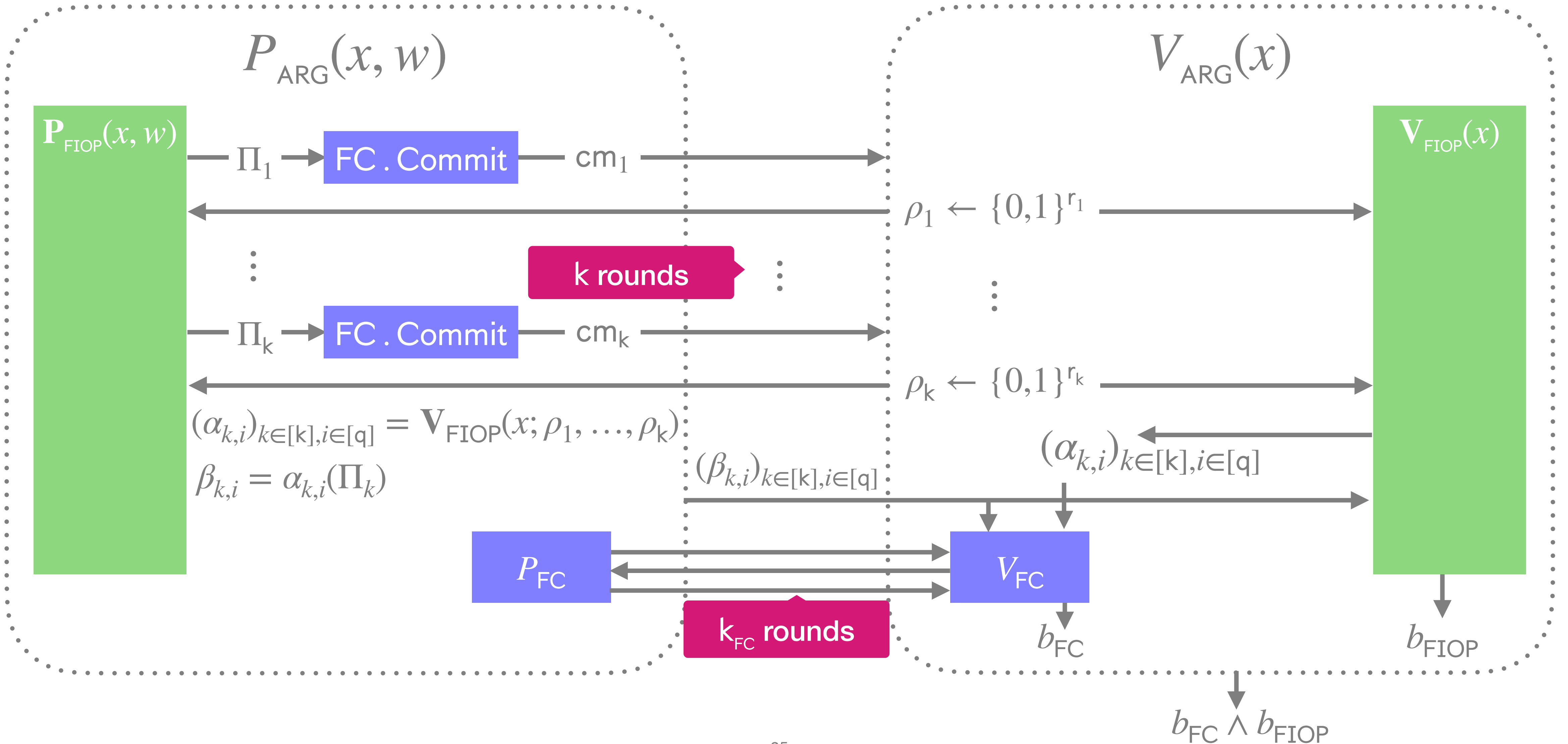
Building block #2: functional interactive oracle proof (FIOP)



Building block #3: functional commitment scheme (FC)



Funky protocol







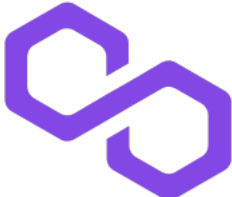




Special cases of the Funky protocol

	Proof string	Query class	Answer
PCP+VC [Kilian92] IOP+VC [BCS16,CDGS23]	$\Pi \in \Sigma^\ell$	point queries $\mathbf{Q}_{\text{point}}$	$\beta = \Pi[\alpha]$ for $\alpha \in [\ell]$
LPCP+LC [LM19]	$\Pi \in \mathbb{F}^\ell$	linear queries \mathbf{Q}_{lin}	$\beta = \sum_{i \in [\ell]} \Pi[i] \cdot \alpha[i]$ for $\alpha \in \mathbb{F}^\ell$
PIOP+PC [CHM+20,BFS20]	$\Pi \in \mathbb{F}[X]^{\leq D}$	evaluation queries on polynomials \mathbf{Q}_{poly}	$\beta = \sum_{i \in [\ell]} \Pi[i] \cdot \alpha^{i-1}$ for $\alpha \in \mathbb{F}$
PIOP*+PC* [GWC19]	$\Pi \in (\mathbb{F}[X]^{\leq D})^{m+n}$ $= (f_1, \dots, f_m, g_1, \dots, g_n)$	evaluation queries on structured polys $\mathbf{Q}_{\text{poly}^*}$	$\beta = \sum_{k \in [n]} h_k(f_1(\alpha), \dots, f_m(\alpha)) \cdot g_k(\alpha)$

Beyond Funky: Bulletproofs (and other sumcheck-based arguments), linear-only encodings [BCIOP13, GGPR13, Groth16], ...

Special cases of the Funky protocol

	Proof string	Query class	Answer
PCP+VC IOP+VC	Funky protocol is everywhere		
LPCP+L	 Succinct	 RISC ZERO	 Ligero
PIOP+P	 STARKWARE	 Aztec	 Matter Labs
PIOP*+I	 polygon	 NEXUS	 Irreducible
			...

Beyond Funky: Bulletproofs (and other sumcheck-based arguments), linear-only encodings [BCIOP13, GGPR13, Groth16], ...

Which security property for FC?

Earlier in this talk **IOP+VC**

$$\epsilon_{\text{ARG}} \approx \epsilon_{\text{IOP}} + \epsilon_{\text{VC}}^{\text{PB}}$$

Vector Commitments

position binding:

$$\Pr \left[\begin{array}{l} \beta_1 \neq \beta_2 \\ \wedge \forall i : \text{FC} . \text{Check}(\text{pp}, \text{cm}, \alpha_i, \beta_i, \text{pf}_i) = 1 \end{array} \middle| (\text{cm}, \alpha, \beta_1, \text{pf}_1, \beta_2, \text{pf}_2) \leftarrow A(\text{pp}) \right] \leq \epsilon$$

[LM19] **LPCP+LC**

$$\epsilon_{\text{ARG}} \approx \epsilon_{\text{LPCP}} + \epsilon_{\text{LC}}^{\text{FB}}$$

Linear Commitments

function binding:

$$\Pr \left[\begin{array}{l} \nexists \Pi : \forall i : \langle \alpha_i, \Pi \rangle = \beta_i \\ \wedge \forall i : \text{FC} . \text{Check}(\text{pp}, \text{cm}, \alpha_i, \beta_i, \text{pf}_i) = 1 \end{array} \middle| (\text{cm}, (\alpha_i, \beta_i, \text{pf}_i)_{i \in [n]}) \leftarrow A(\text{pp}) \right] \leq \epsilon$$

[CHMMVW20, BFS20] **PIOP+PC**

$$\epsilon_{\text{ARG}} \approx \epsilon_{\text{PIOP}} + \kappa_{\text{PC}}$$

Polynomial Commitments

binding? strong correctness? interpolation binding? extractability?

[KZG10]

[AJMMS23]

[CHM+20, BFS20]

Too strong

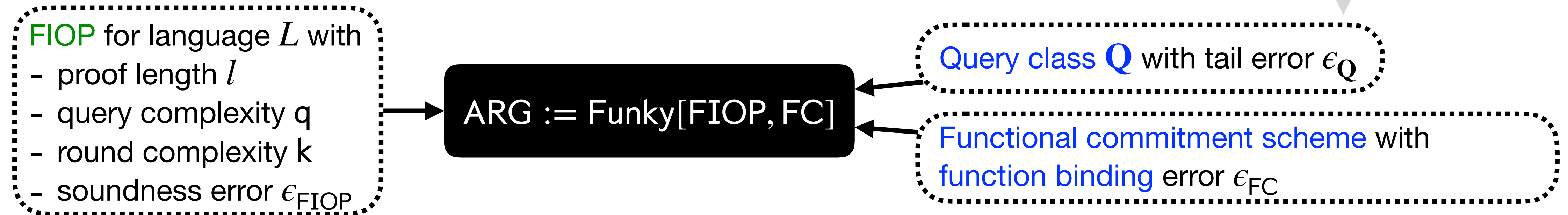
Functional Commitments

function binding:

$$\Pr \left[\begin{array}{l} \nexists \Pi : \forall i : \alpha_i(\Pi) = \beta_i \\ \wedge \forall i : \text{FC} . \text{Check}(\text{pp}, \text{cm}, \alpha_i, \beta_i, \text{pf}_i) = 1 \end{array} \middle| (\text{cm}, (\alpha_i, \beta_i, \text{pf}_i)_{i \in [n]}) \leftarrow A(\text{pp}) \right] \leq \epsilon$$

Security of Funky protocol

Internal property of \mathbf{Q}
Independent of FIOP/FC



Theorem. $\forall N \in \mathbb{N}$,

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{FIOP}} + k \cdot \epsilon_{\text{FC}}(t_{\text{FC}}) + k \cdot \epsilon_{\mathbf{Q}}(l, N), \text{ where } t_{\text{FC}} = O(t_{\text{ARG}} \cdot N).$$

TLDR:

- A “tight” security notion for FC schemes
- Concrete and tight bounds using tail errors

$\epsilon_{\mathbf{Q}_{\text{point}}}(l, N) = l/N \implies$ recovers the bounds for Kilian’s protocol and IBCS protocol

Fiat-Shamir security: From succinct arguments to SNARGs

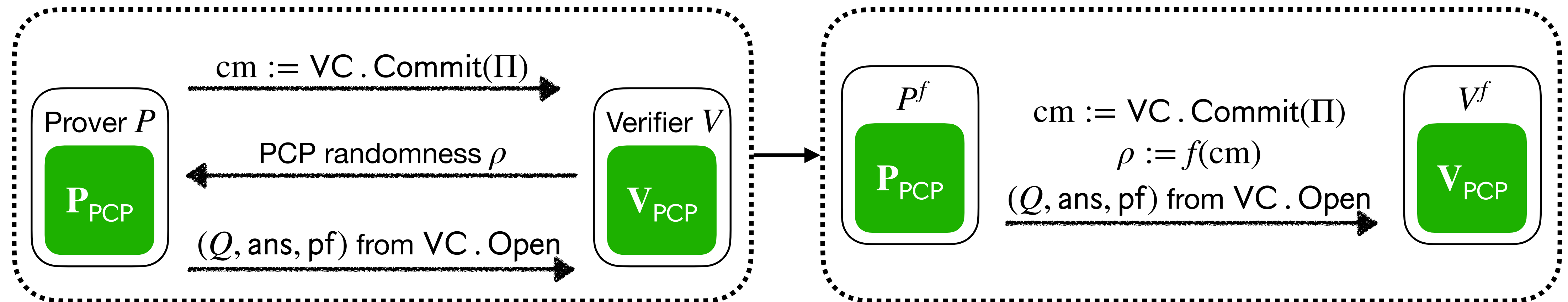
Fiat-Shamir transformation

Random oracle: $\mathcal{O} = \{\mathcal{O}_\lambda\}_{\lambda \in \mathbb{N}}$

\mathcal{O}_λ : uniform distribution over $\{f: \{0,1\}^* \rightarrow \{0,1\}^\lambda\}$

Succinct interactive arguments

Succinct non-interactive arguments



Central question: Is security preserved after the Fiat-Shamir transformation?

In generical **no** [CY24]: $\epsilon_{NARG}(x, t, m) \leq (m+1)^k \cdot \epsilon_{ARG}(x, t)$

RO queries

k might be superconstant!

Fiat-Shamir security



Theorem. $\forall N \in \mathbb{N}$,

$$\epsilon_{\text{NARG}}(t_{\text{ARG}}, m_{\text{ARG}}) \leq \epsilon_{\text{FIOP}}^{\text{FS}}(O(m_{\text{ARG}})) + k \cdot \epsilon_{\text{FC}}^{\text{FSFB}}(t_{\text{FC}}, m_{\text{FC}}) + k \cdot \epsilon_Q(l, N), \text{ where } \begin{cases} t_{\text{FC}} = O(t_{\text{ARG}} \cdot N) \\ m_{\text{FC}} = O(m_{\text{ARG}} \cdot k \cdot N) \end{cases}.$$

A theorem that generalizes everything we saw (except post-quantum)

Corollary: security analysis of Plonk [GWC19] from falsifiable assumption



...

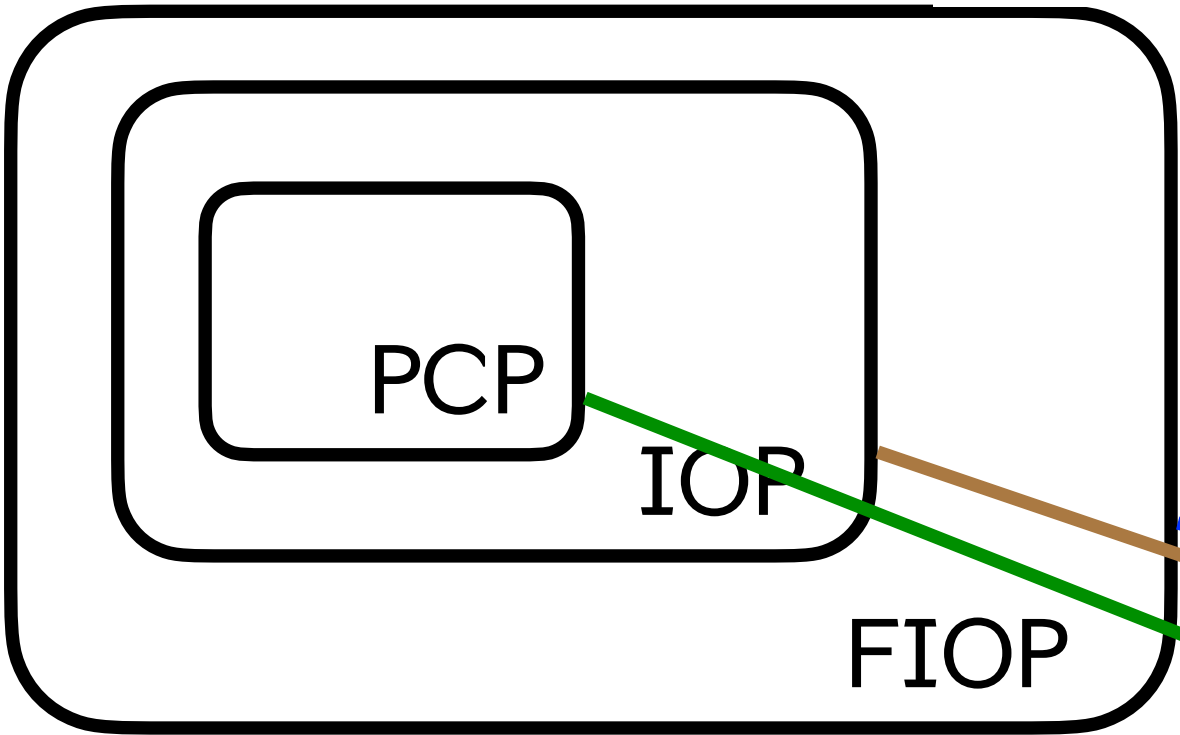
Open problems

SNARGs for NP via Fiat–Shamir in the Plain Model

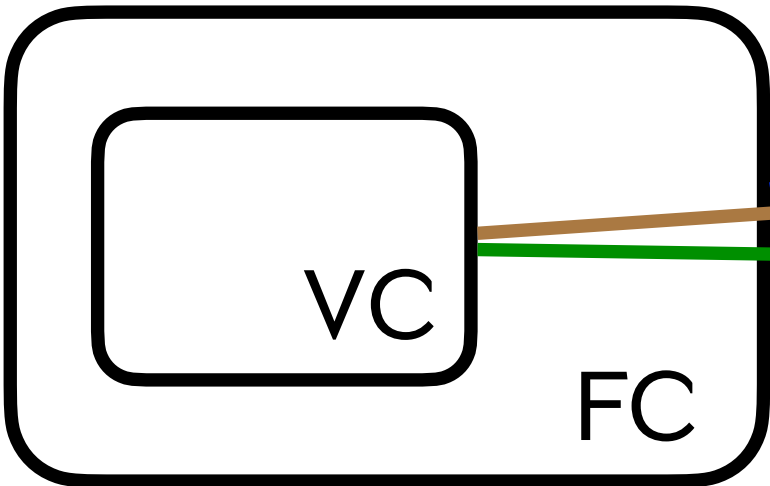
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Probabilistic proofs



Commitment schemes



- Funky protocol**
- Soundness
 - Fiat-Shamir soundness

- IBCS protocol**
- Soundness
 - Private-coin IOPs
 - Post-quantum soundness

- Kilian's protocol**
- Soundness
 - Lower bounds on soundness

Standard model Fiat-Shamir wo/ iO?

Quantum analogue?

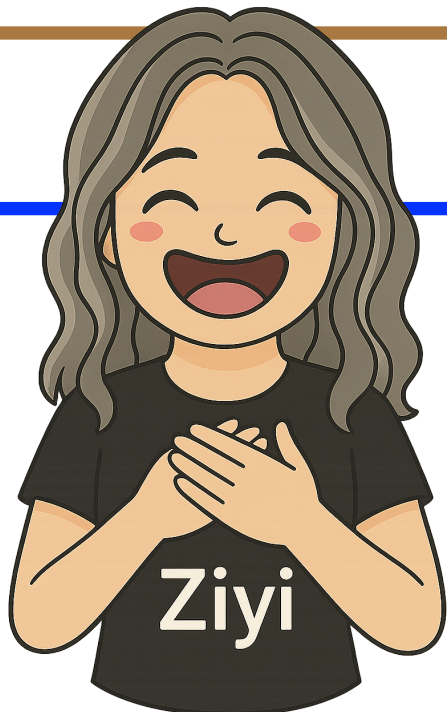
Post-quantum security?

Expected-time regime?

Practical security in idealized models?

Public-query IOPs?

Kilian vs. Sigma protocols?



Ziyi

Thank you!

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