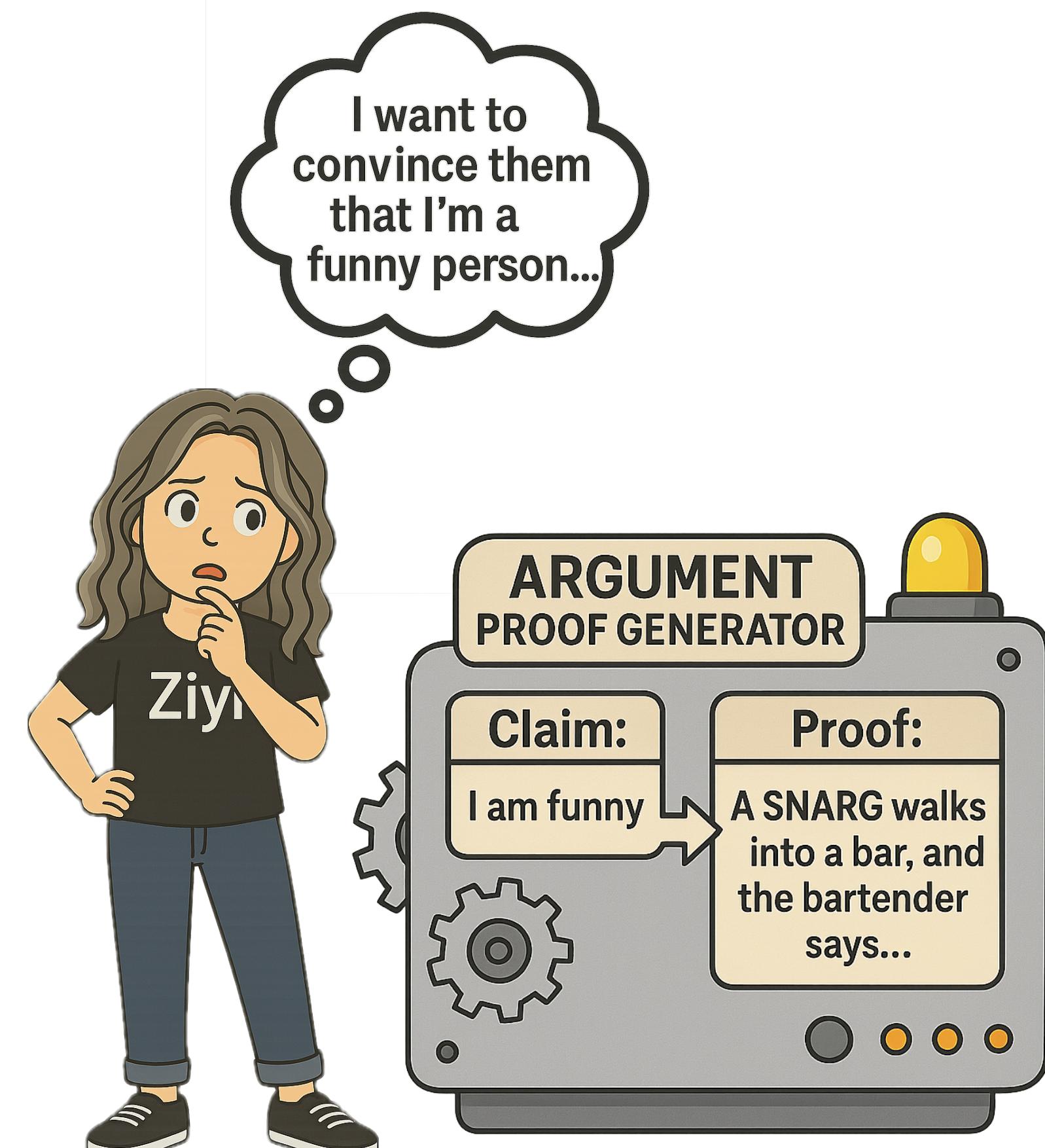


# On the Security of Succinct Arguments from Probabilistic Proofs



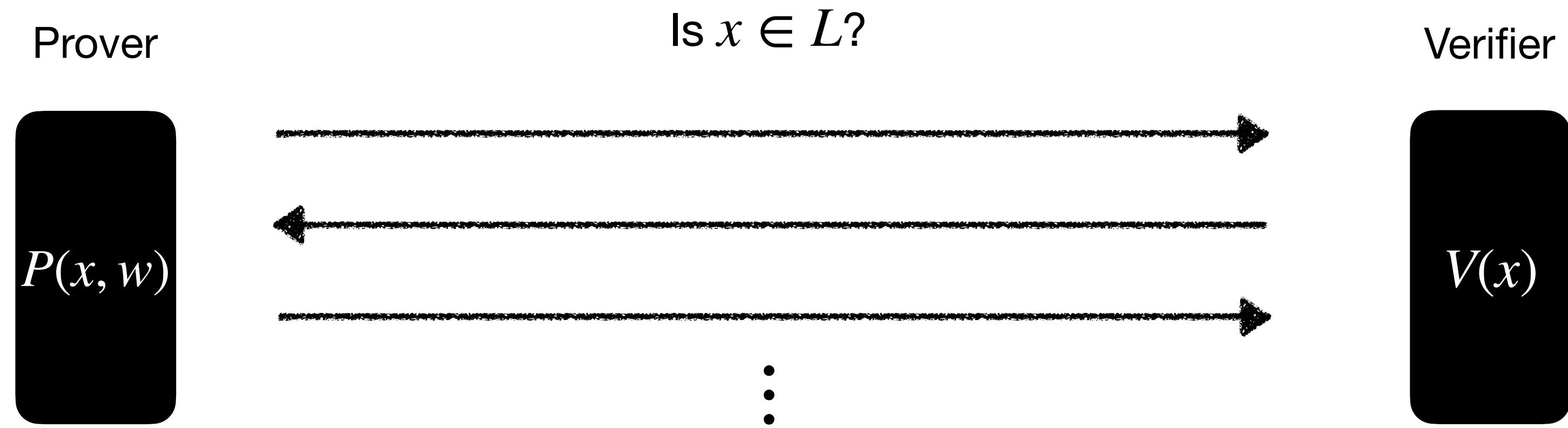
Ziyi Guan



EPFL

# What are succinct arguments?

# Interactive proofs



**Completeness:**  $\forall x \in L, \Pr [\langle P(x, w), V(x) \rangle = 1] = 1$

**Soundness:**  $\forall x \notin L$  and adversary  $\tilde{P}, \Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon$

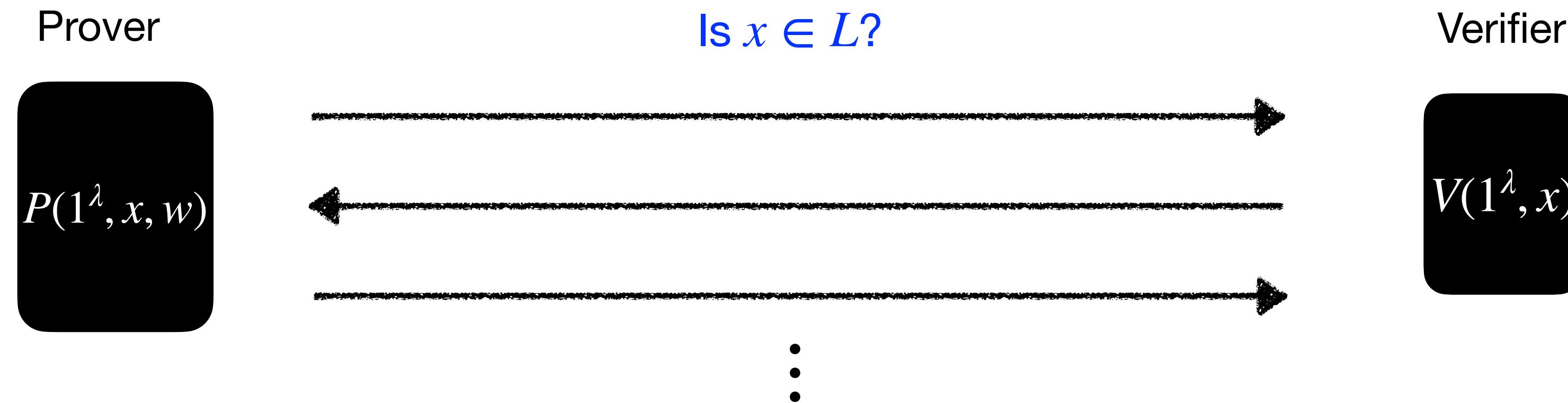
**Target metric:** **COMMUNICATION COMPLEXITY**

**Limitation:** NP-complete languages do not have IPs with  $\mathbf{CC} \ll |w|$

[GH97]:  $\mathbf{IP[CC]} \subseteq \mathbf{BPTIME[2^{CC}]}$

# Interactive arguments

Interactive proofs with computational soundness



**Computational soundness:**  $\forall x \notin L$  and  $t_{\text{ARG}}$ -time adversary  $\tilde{P}$ ,  $\Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon_{\text{ARG}}(t_{\text{ARG}})$

**AMAZING:**  $\exists$  interactive arguments for NP with  $\mathbf{CC} \ll |w|$  (given basic cryptography)

Today's protagonist:  
**Succinct Interactive Arguments**

$cc \ll |w|$

# Why study **succinct** interactive arguments?

$\text{time}(V) \ll |w|$

They exist based on simple crypto assumptions...

... so they play a role in numerous cryptotheory results.

zero-knowledge with  
non-black-box simulation

malicious MPC

...

They are a stepping stone for SNARGs, which have numerous real-world applications.

 **Succinct**

 **RISC ZERO**

 **Aztec**

 **VALIDA**

**Irreducible**

 **STARKWARE**

 **polygon**

 **NEXUS**

 **Ligero**

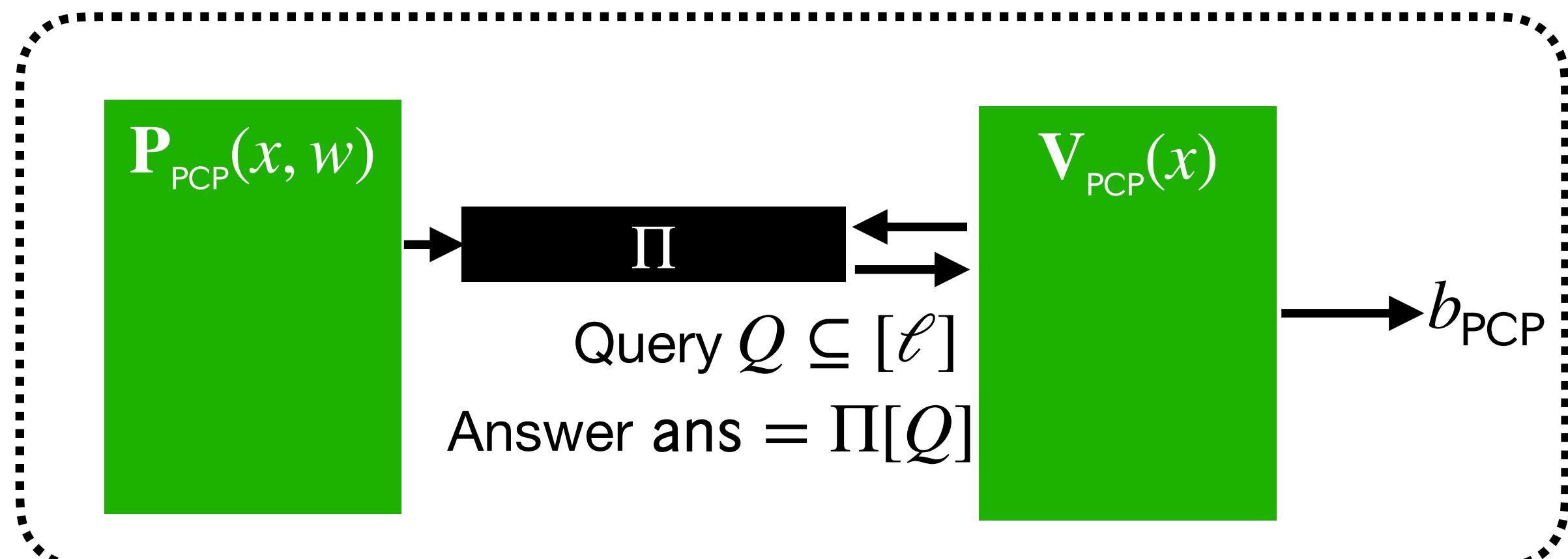
 **MatterLabs**

...

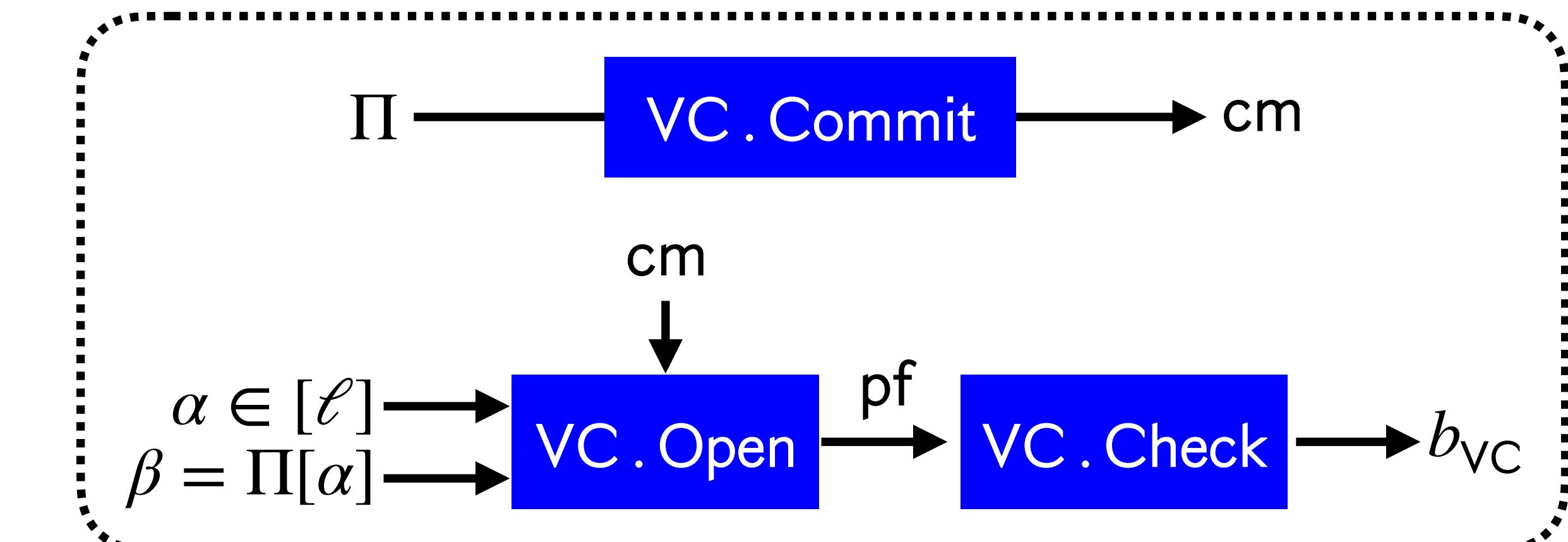
# **Kilian's protocol: The first and simplest succinct argument**

# How to construct succinct arguments?

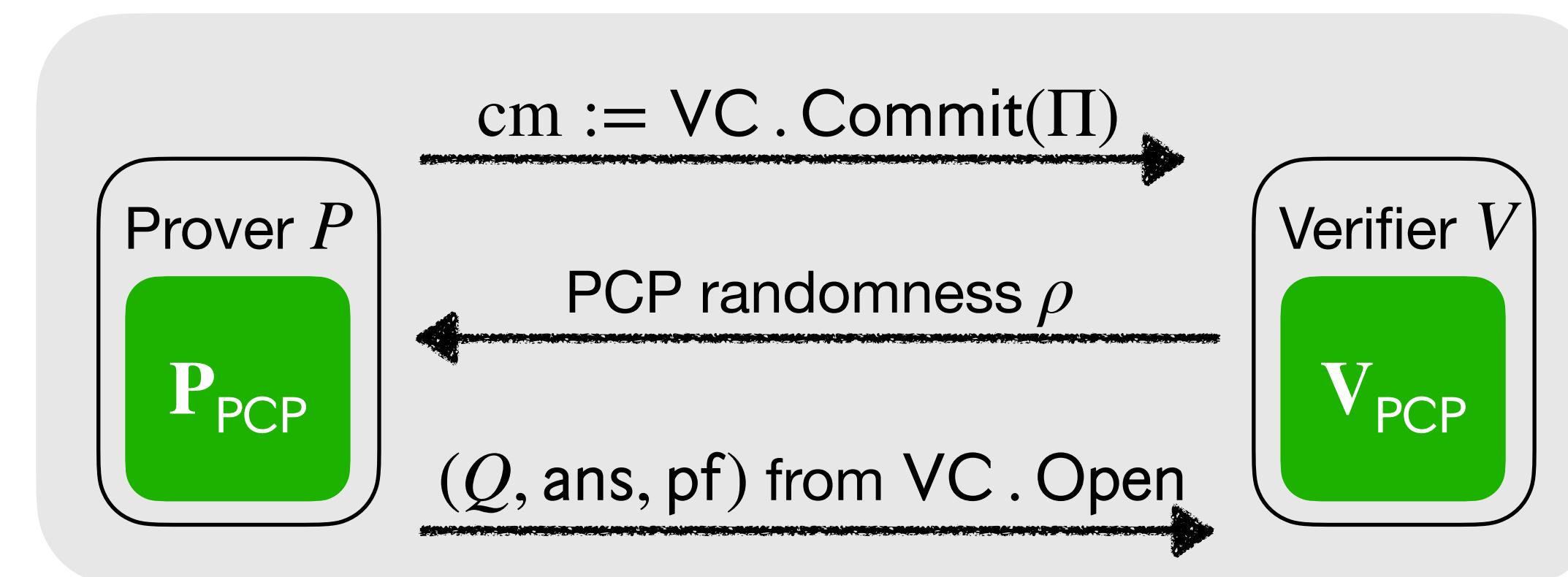
**Building block #1:** probabilistically checkable proof (PCP)



**Building block #2:** vector commitment scheme (VC)



## Kilian's protocol

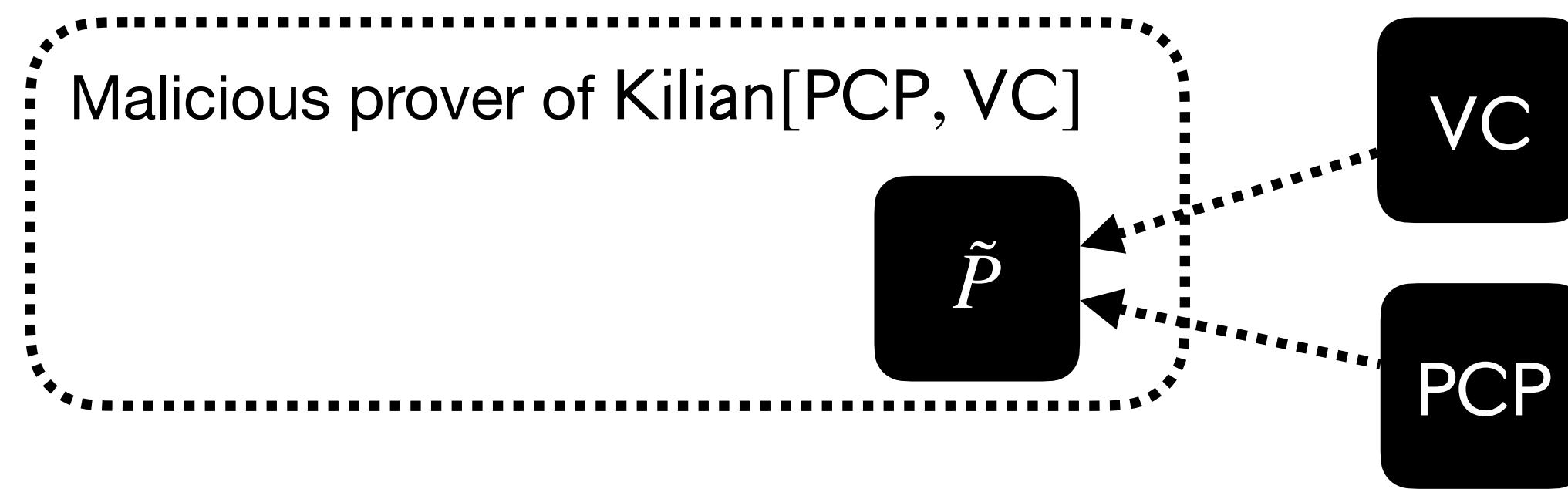


$T$ -step computation:

- Prover time:  $\text{poly}(T)$
- Verifier time:  $\text{polylog}(T)$
- (**CC**:  $\text{polylog}(T)$ )

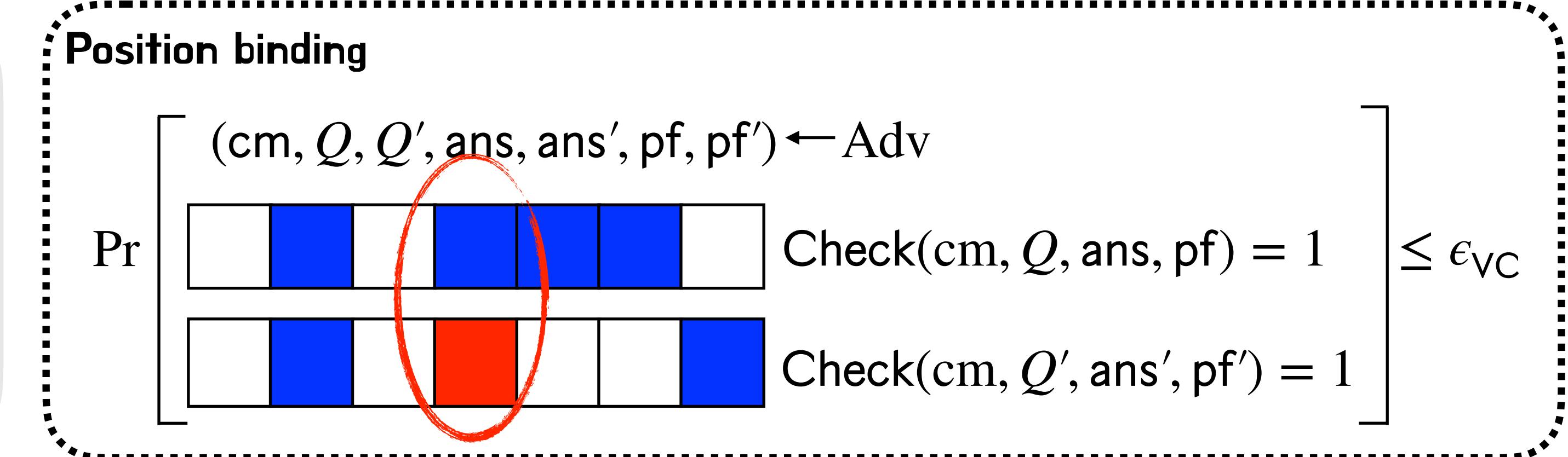
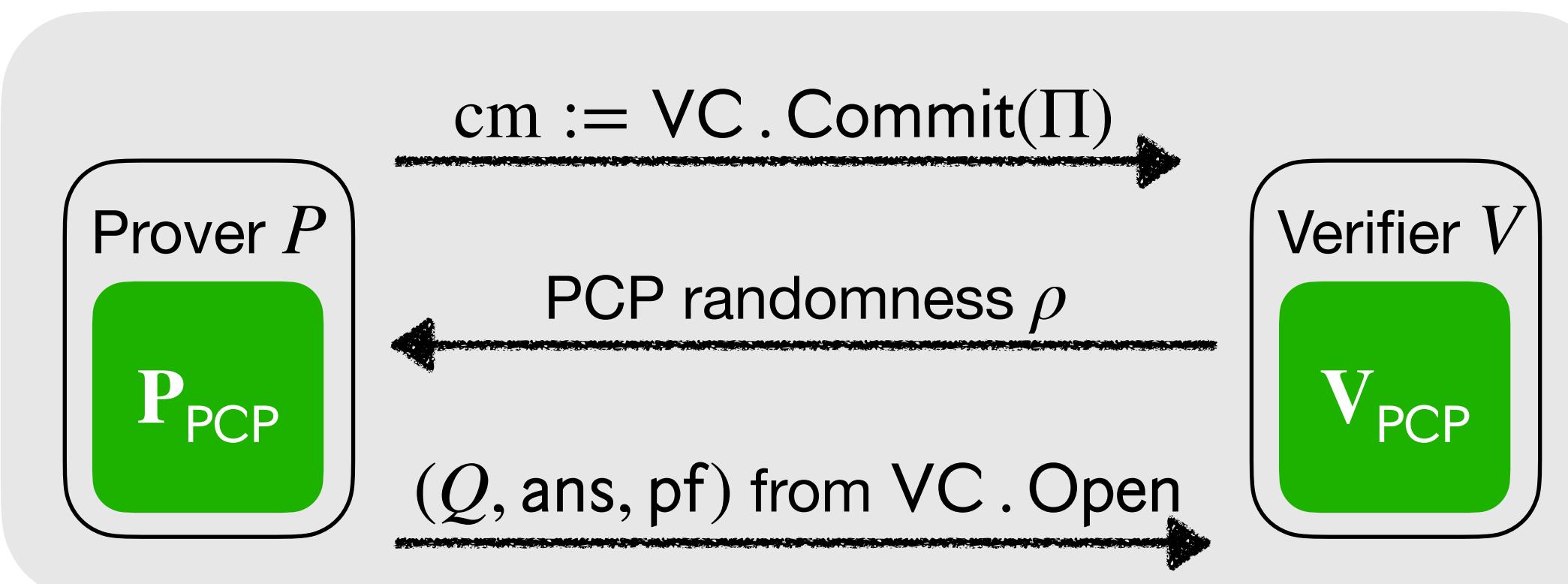
# Simple (and only known) security analysis

**Goal:** relate the soundness error of Kilian[PCP, VC]  
to the soundness error of PCP and the position binding error of VC.



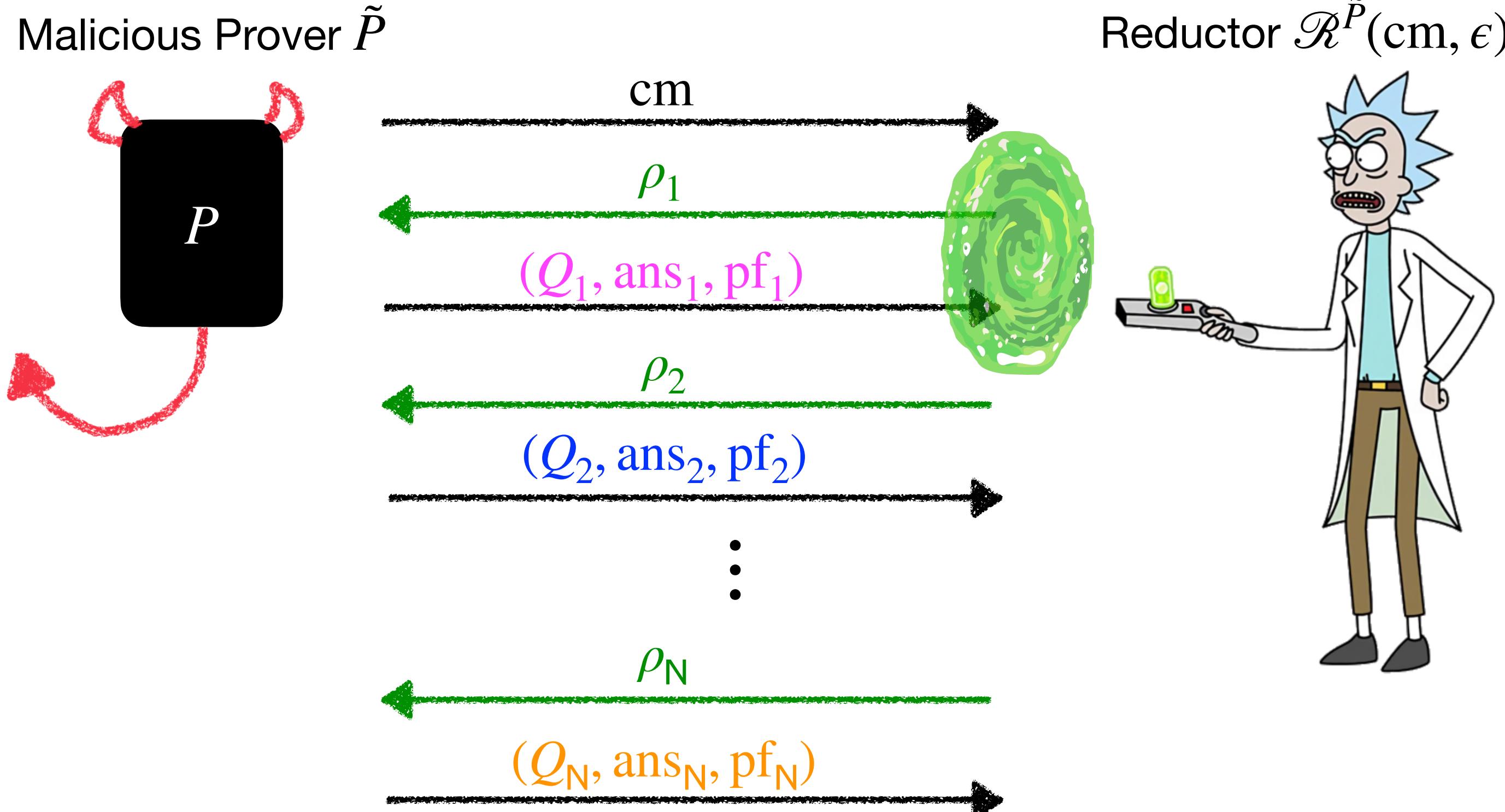
**Rewind**  $\tilde{P}$  to get a malicious PCP string  $\tilde{\Pi}$   
⇒ (PCP soundness) upper bound the success probability of  $\tilde{\Pi}$   
⇒ (Position binding)  $\tilde{\Pi}$  cannot be too different from  $\tilde{P}$

## Kilian's protocol



# Security from rewinding

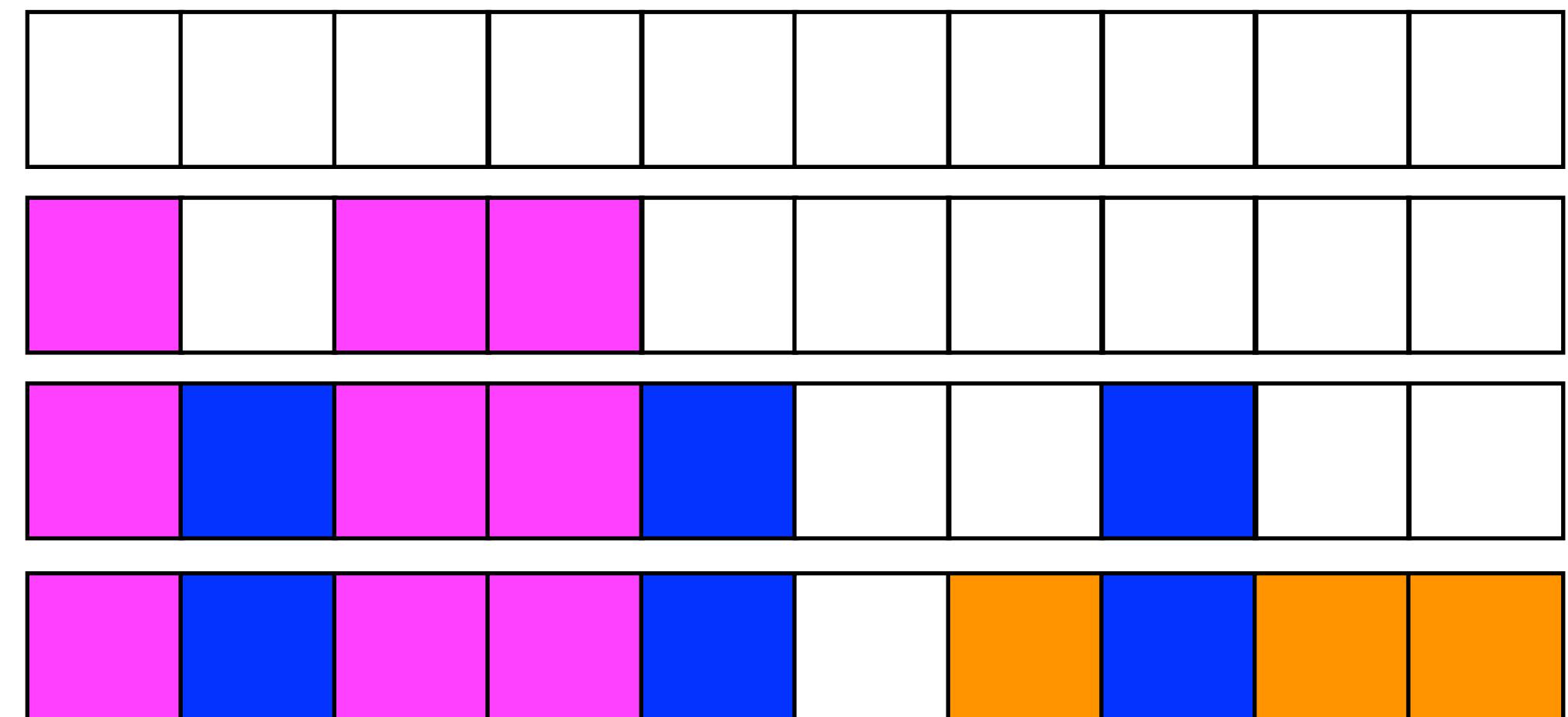
## How to rewind?



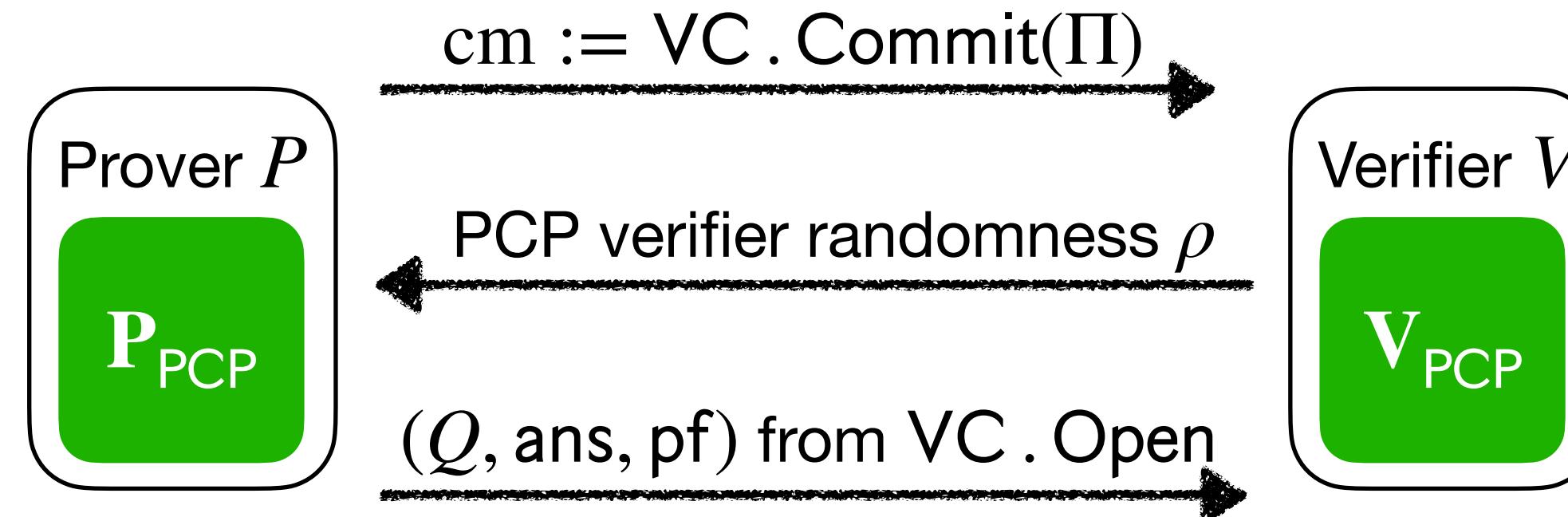
Subtle design choices:

- Strict time vs. expected time
- Sample with/without replacement
- Stopping conditions
- ...

Recover  $\tilde{\Pi}$



# What is the security of Kilian's protocol?



**Previously:**

- [Kilian92] gives an **informal** analysis
- [BG08]  $\epsilon_{\text{ARG}} \leq 8 \cdot \epsilon_{\text{PCP}} + \sqrt[3]{\epsilon_{\text{VC}}}$  and **assuming** PCP is **non-adaptive** & **reverse-samplable**
- [CMSZ21] Kilian is secure when  $\epsilon_{\text{PCP}}$  **negligible** (in a paper about post-quantum security)

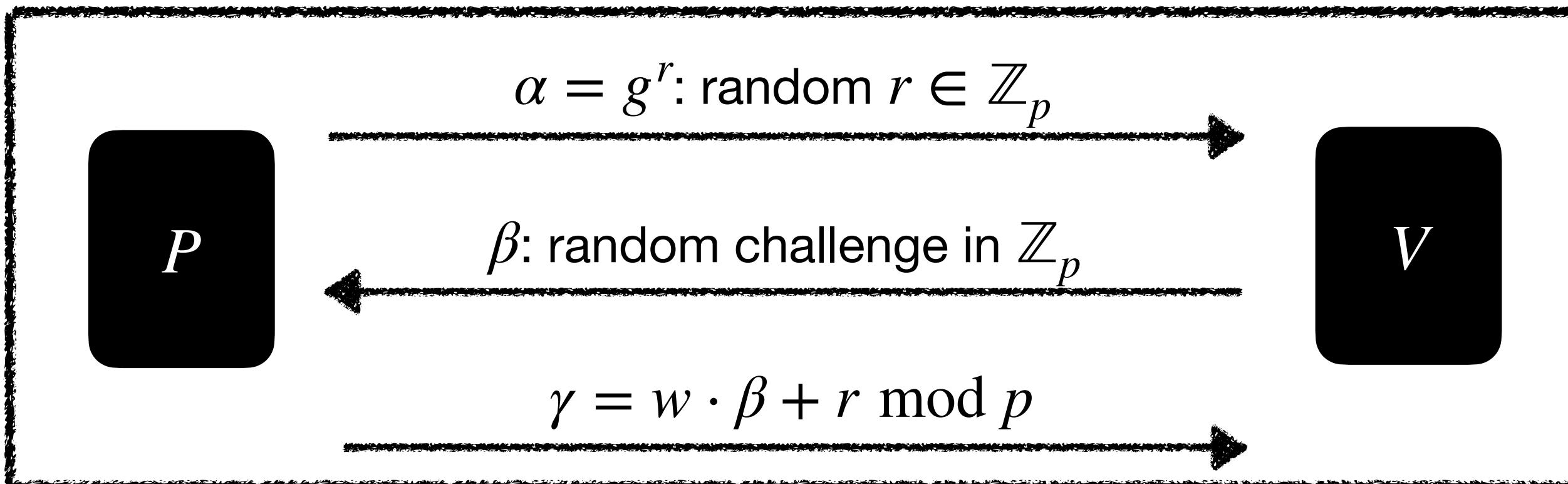
non-trivial restrictions

We expect that  $\epsilon_{\text{ARG}} \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}} \dots$  right?



# Surprise! A limitation:

$$\epsilon_{\text{ARG}} \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}} \implies \text{breakthrough on Schnorr}$$



Lots of work on Schnorr security  
[Sho97,PS00,BP02,FPS20,BD20,RS21,SSY23] ...  
and yet there are still open questions on its optimal security!

**Theorem.**  $\exists$  PCP and VC s.t.

$$\epsilon_{\text{Schnorr}}(t) \leq \epsilon_{\text{ARG}}(t).$$

Similar bound holds for  
expected-time adversary

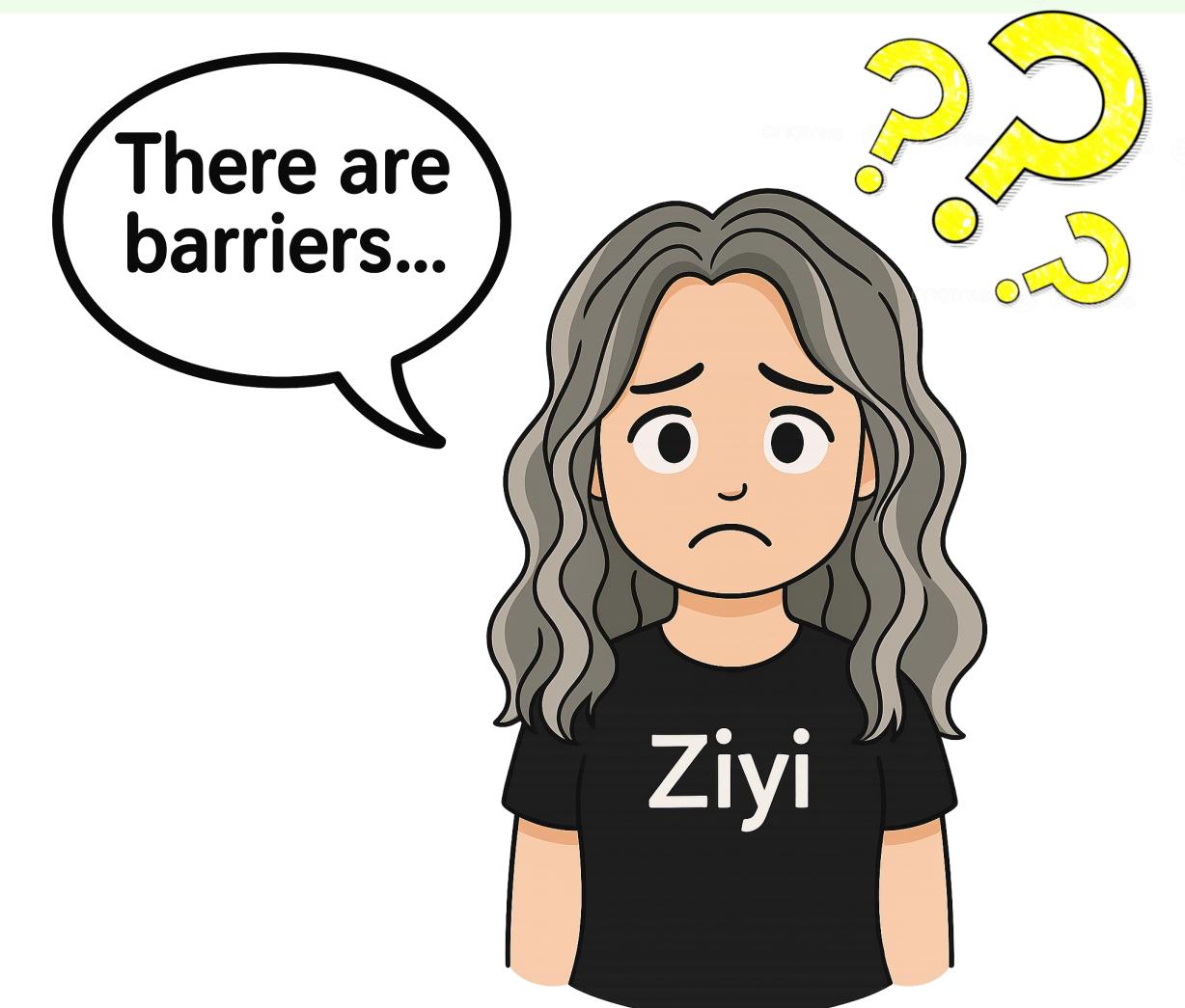
Suppose  
 $\epsilon_{\text{ARG}} \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}$



$$\epsilon_{\text{Schnorr}}(t_{\text{Schnorr}}) \leq \epsilon_{\text{DLOG}}(O(t_{\text{Schnorr}}))$$

Best analysis of Schnorr [PS00]:  $\epsilon_{\text{Schnorr}}(t_{\text{Schnorr}}) \leq \sqrt{\epsilon_{\text{DLOG}}(O(t_{\text{Schnorr}}))}$

... so the folklore is beyond current rewinding techniques



# Improved security for Kilian



**Theorem.**  $\forall \epsilon > 0$ ,

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot l \cdot 1/\epsilon).$$

**Why  $l \cdot 1/\epsilon$  overhead?**

- $l$  locations in  $\Pi$
- $\implies$  Rewind at least  $l$  times (e.g. maybe all PCP queries but 1 are fixed)
- Some rewinds yield garbage:
  - The locations were already found
  - VC check fails
- $\implies$  Need  $1/\epsilon$  times for each location as buffer

$\lambda$ : security parameter

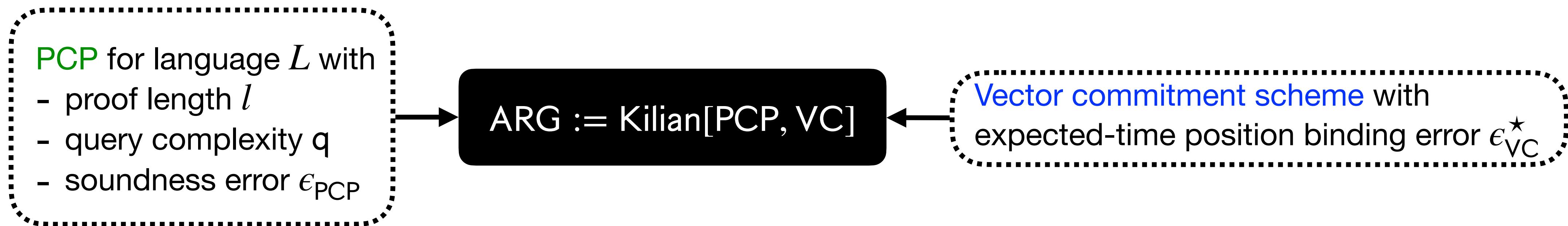
Suppose  $\epsilon_{\text{VC}}(t) \leq O(t^2/2^\lambda)$  (e.g. an ideal Merkle tree)

By **Theorem**:

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{PCP}} + l^{2/3} \cdot O\left(\sqrt[3]{t_{\text{ARG}}^2/2^\lambda}\right)$$

**That is,**  $\epsilon_{\text{ARG}} \leq \epsilon_{\text{PCP}} + \sqrt[3]{\epsilon_{\text{VC}}}$

# Alternative route: expected-time regime



**Theorem.**  $\forall \epsilon > 0$ ,

$$\epsilon_{\text{ARG}}^*(t_{\text{ARG}}^*) \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}^*(t_{\text{VC}}^*) + \epsilon, \text{ where } t_{\text{VC}}^* = O(t_{\text{ARG}}^* \cdot \log(q/\epsilon)).$$



Set  $\epsilon_{\text{VC}}^*(t^*) \leq O\left(\sqrt{(t^*)^2/2^\lambda}\right)$   $\lambda$ : security parameter  
(e.g. an ideal Merkle tree)

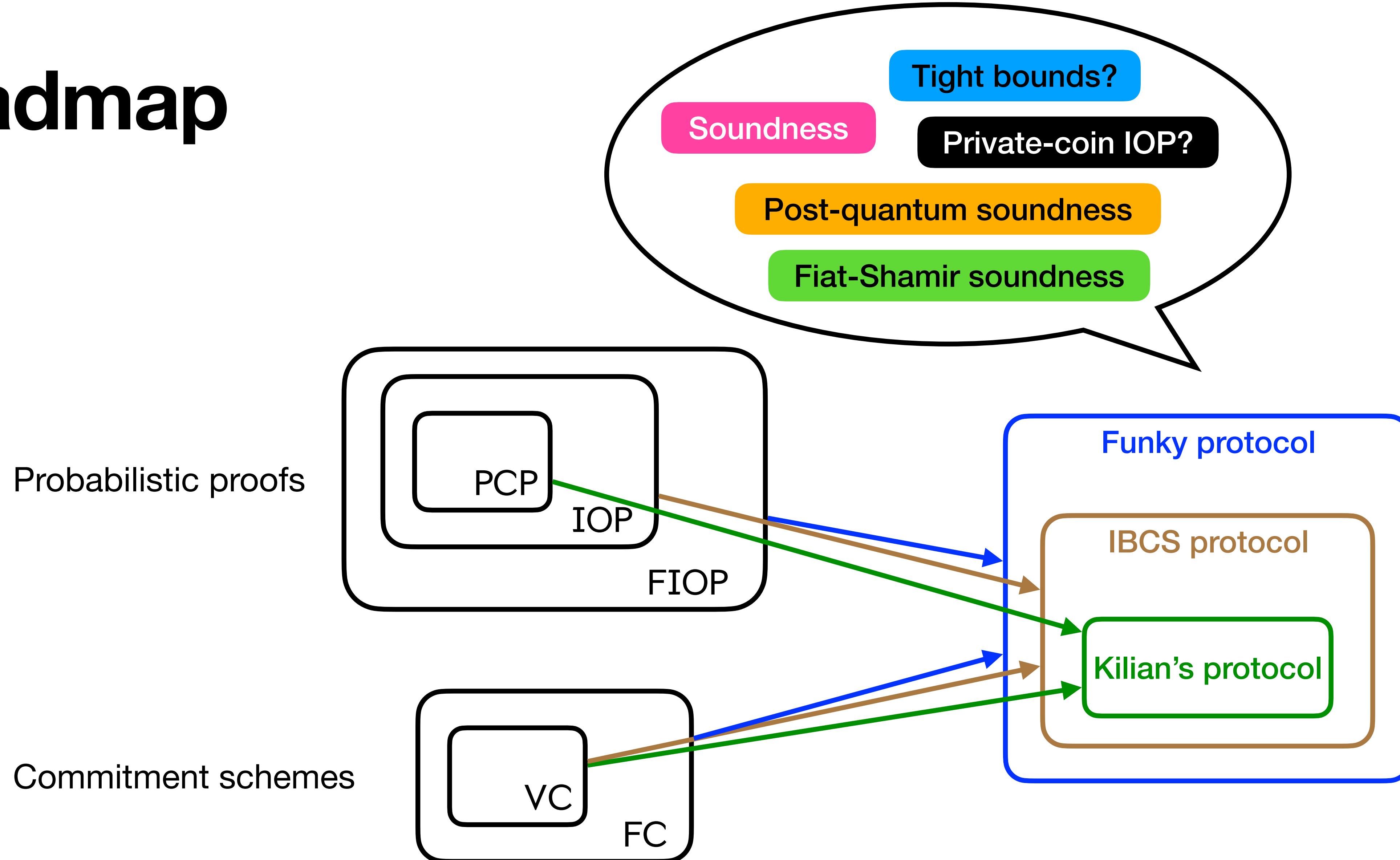
**Theorem**  
 $\implies$

$$\begin{aligned} \epsilon_{\text{ARG}}^*(t_{\text{ARG}}^*) &\leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}^*(t_{\text{ARG}}^* \cdot \log(q/\epsilon)) + \epsilon \\ &\leq \epsilon_{\text{PCP}} + \text{polylog}\left(q \cdot \sqrt{(t_{\text{ARG}}^*)^2/2^\lambda}\right) \cdot O\left(\sqrt[2]{(t_{\text{ARG}}^*)^2/2^\lambda}\right) \end{aligned}$$

small factor

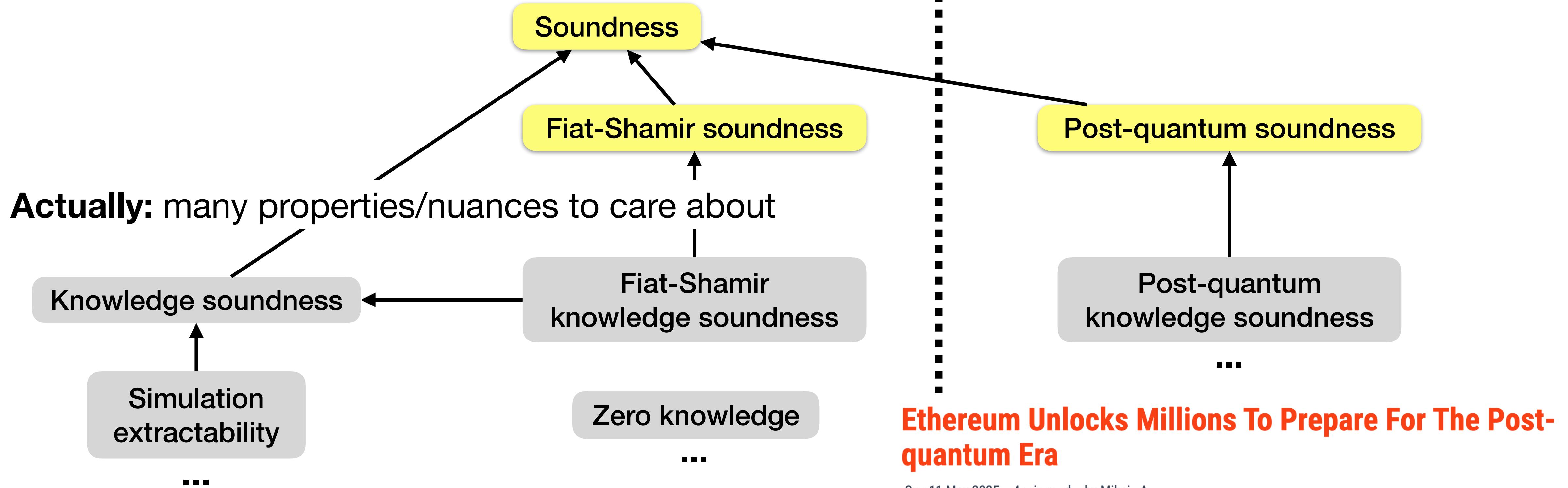
We achieved  $\epsilon_{\text{ARG}}^* \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}^*$  !

# Roadmap



# On security notions of arguments

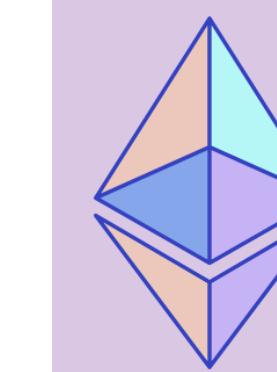
**Today:** focus on soundness only



Sun 11 May 2025 • 4 min read • by Mikaia A.

Strict-time adversary  
vs.  
Expected-time adversary

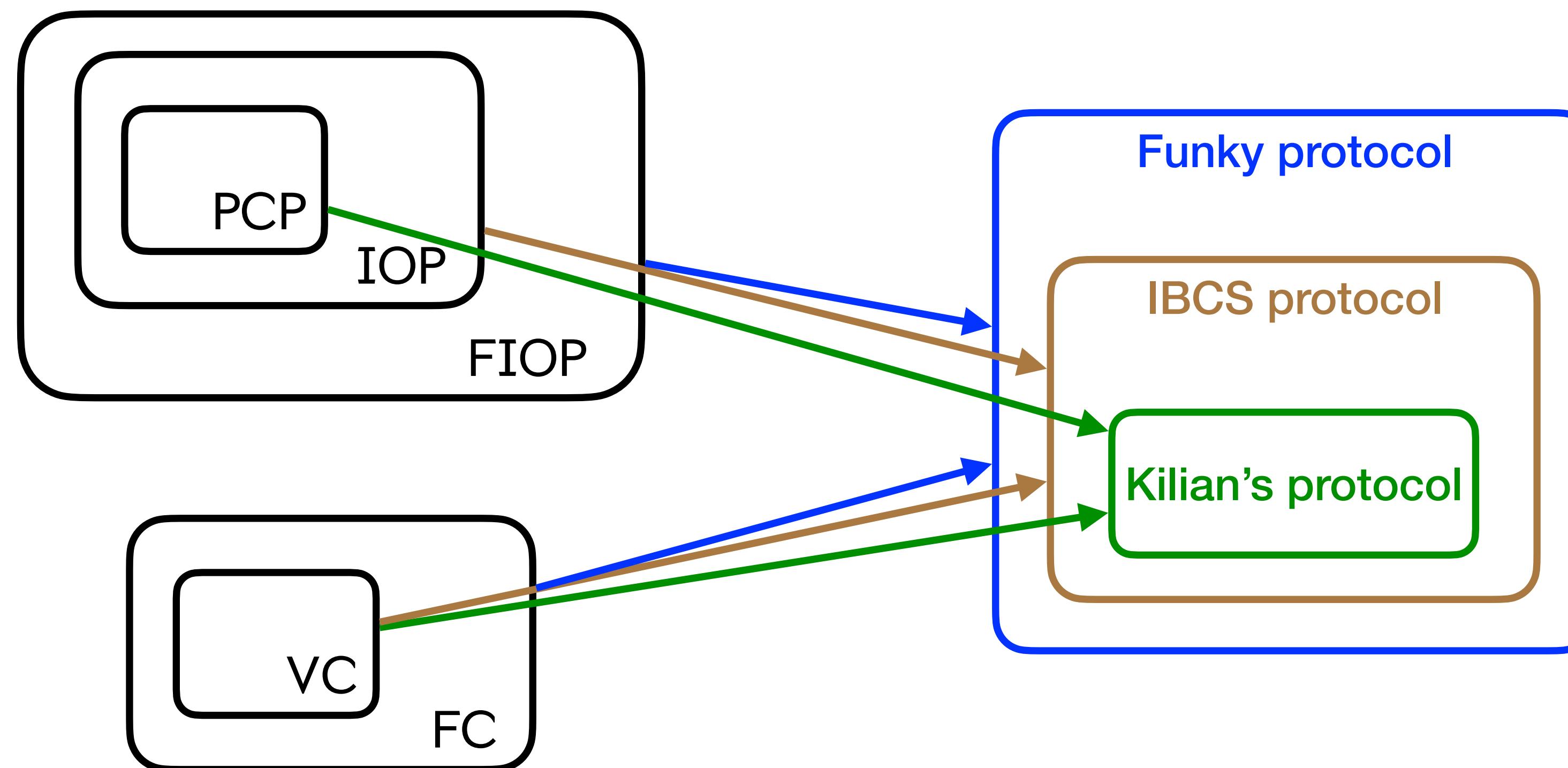
e.g., [BL02] zero-knowledge  
protocols do not have strict  
poly-time (black-box) extractor



**zkEVM Formal Verification Project**

A project by the Ethereum Foundation to accelerate the application of formal  
verification methods to zkEVMS

# IBCS protocol: Using IOPs instead of PCPs

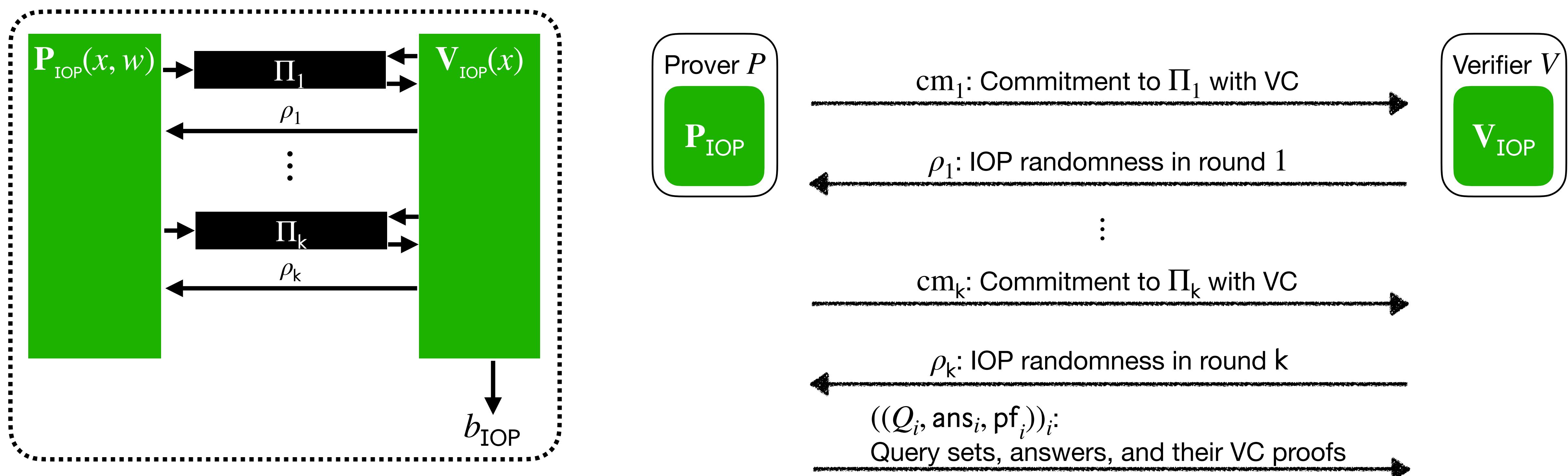


# IBCS protocol

Existing PCPs are not concretely efficient: prover time too big

People use IOPs

Public-coin interactive oracle proof (IOP)



# Security of IBCS protocol

Public-coin IOP for language  $L$  with

- proof length  $l$
- query complexity  $q$
- round complexity  $k$
- soundness error  $\epsilon_{\text{IOP}}$

ARG := IBCS[IOP, VC]

Vector commitment scheme with  
position binding error  $\epsilon_{\text{VC}}$

The ideal bound  $\epsilon_{\text{ARG}} \leq \epsilon_{\text{IOP}} + \epsilon_{\text{VC}}$  is not possible... What can we get?

**Theorem.**  $\forall \epsilon > 0$ ,

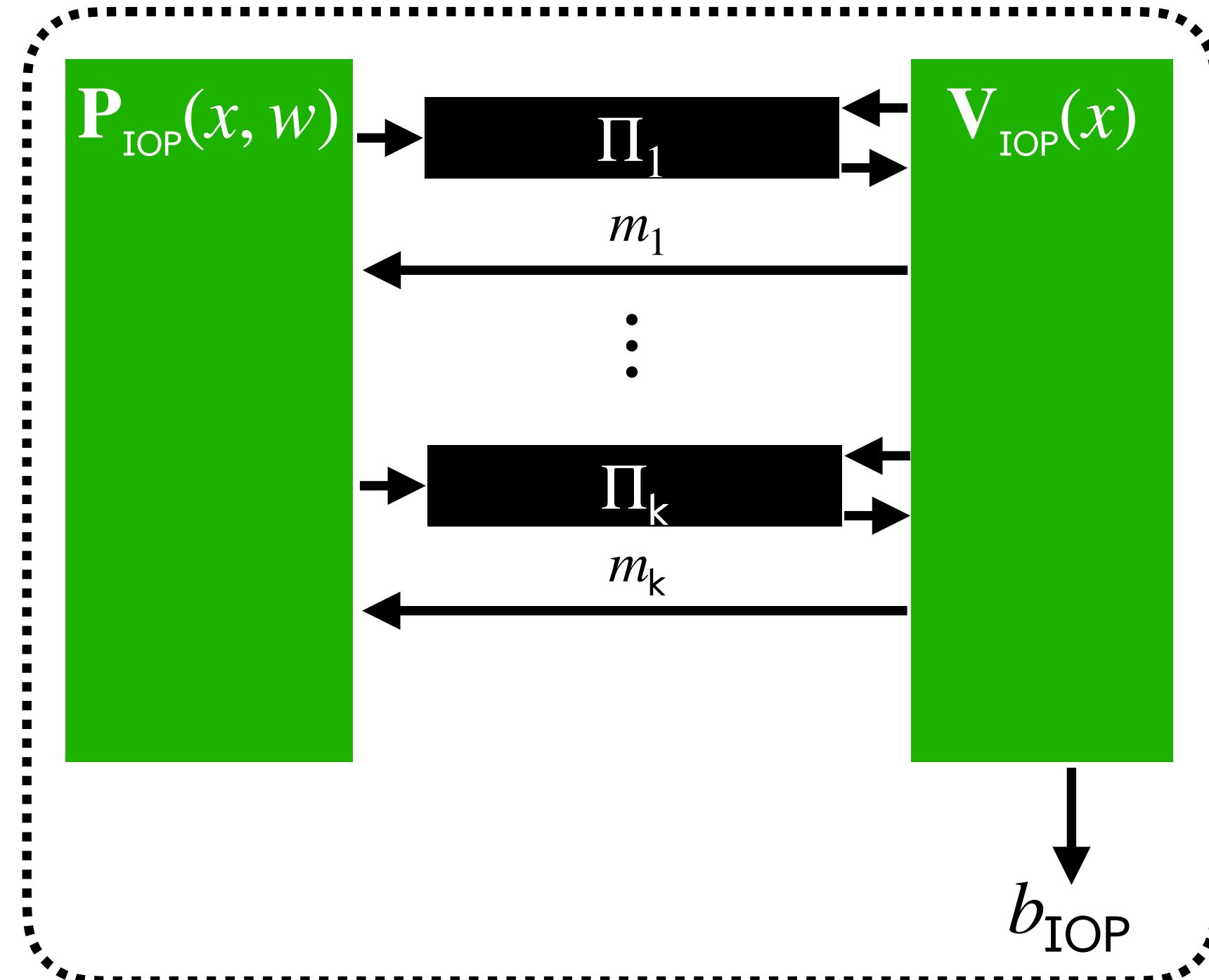
$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + \textcolor{red}{k \cdot \epsilon_{\text{VC}}(t_{\text{VC}})} + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot l/\epsilon).$$

Recall, for Kilian's protocol:  $\forall \epsilon > 0$ ,

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{PCP}} + \textcolor{red}{1 \cdot \epsilon_{\text{VC}}(t_{\text{VC}})} + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot l/\epsilon).$$

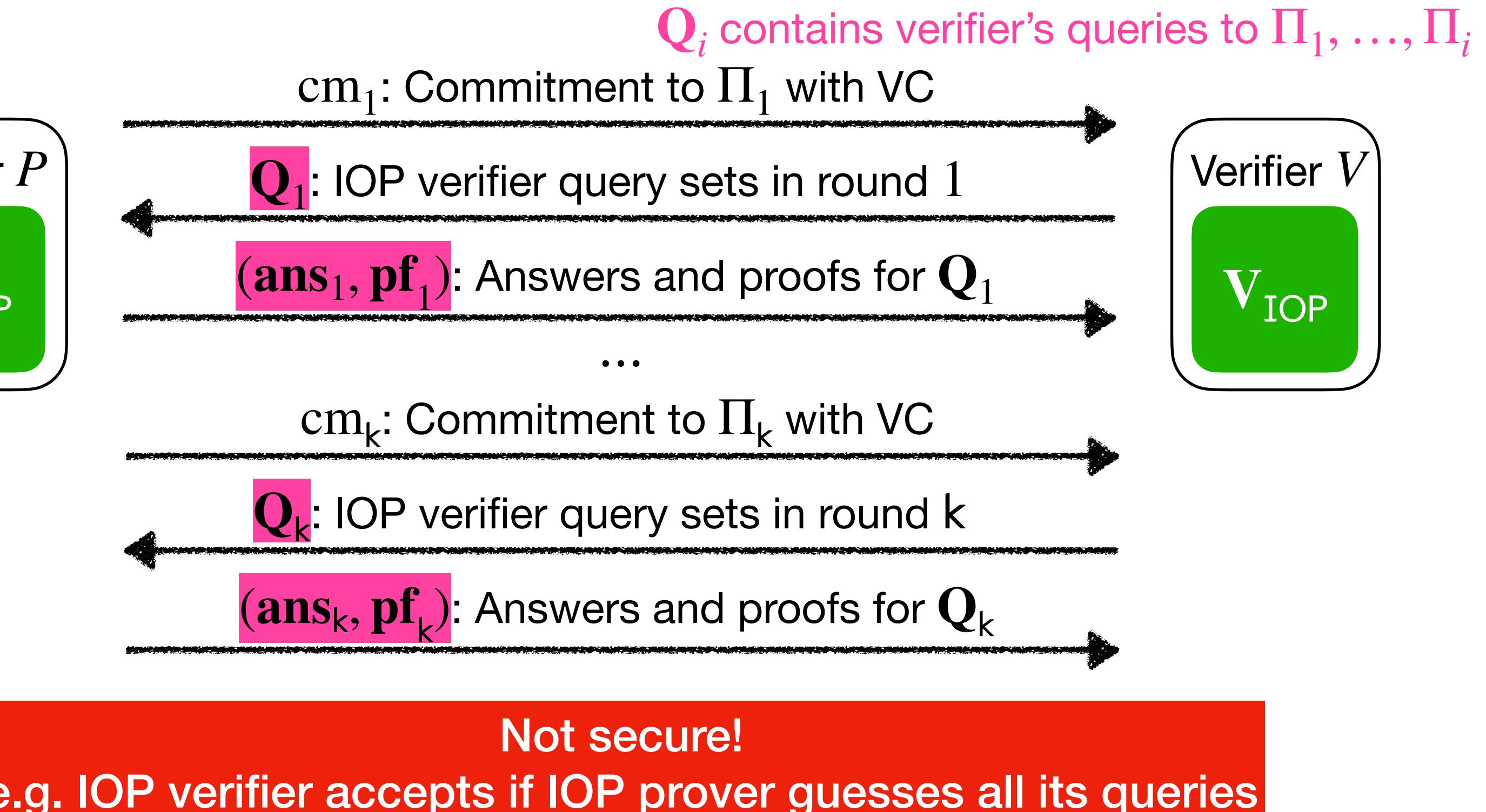
# Why do we need public-coin IOPs?

Private-coin interactive oracle proof (IOP)



How about public-query IOPs?

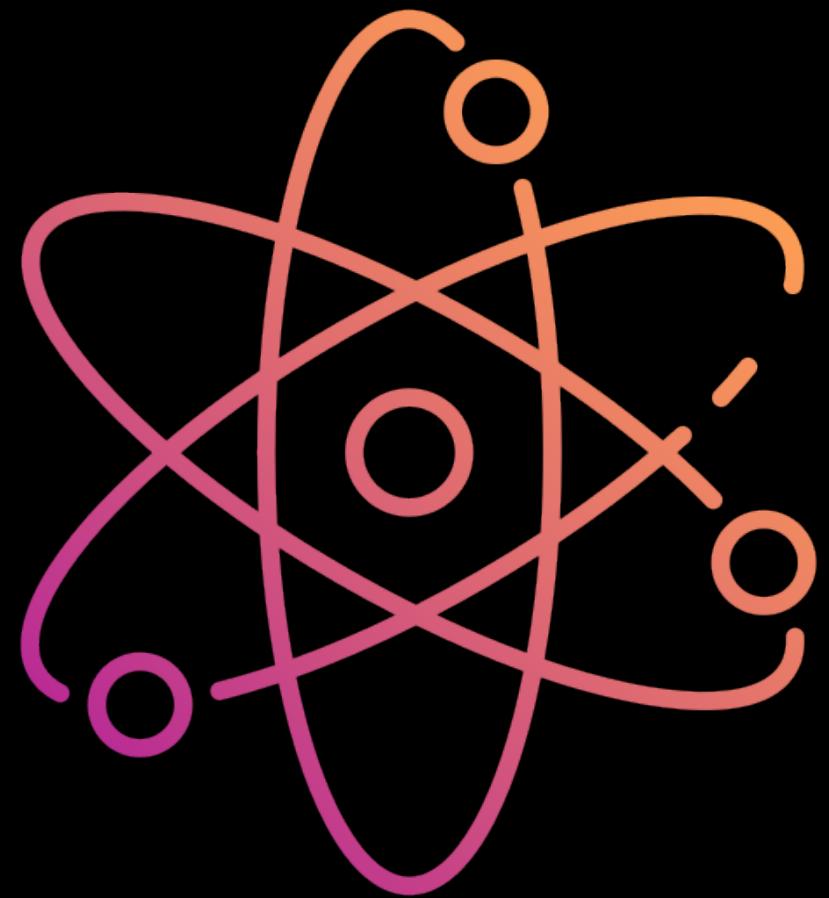
Queries can be learned by the prover (in "real-time")



Clearly, the IBCS protocol is secure whenever the underlying IOP is public-query... right?

**Lemma:** secure if IOP has an "efficient random continuation sampler"

**Open question:** can we prove security for ALL public-query IOPs?  
(Or maybe there is a black-box barrier?)

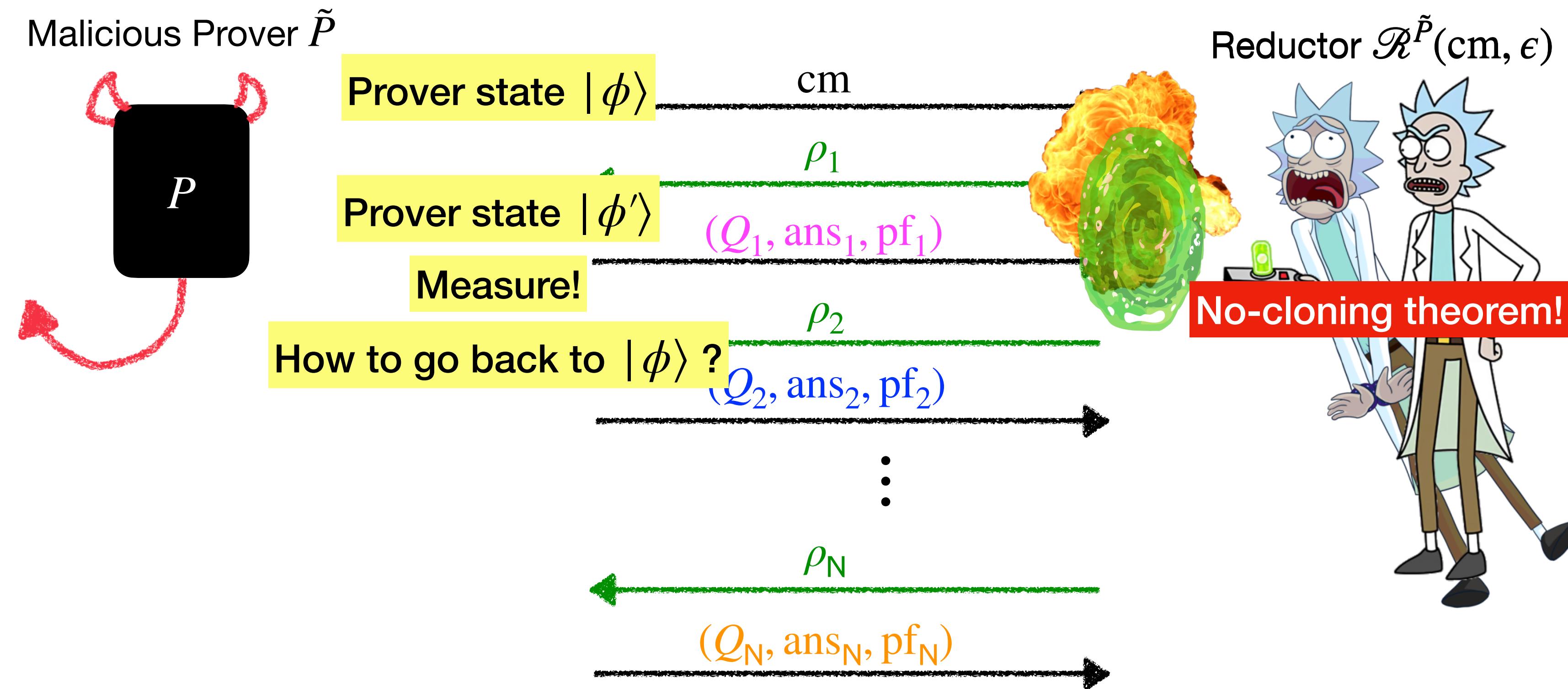


## Interlude: **post-quantum** security

**Post-quantum soundness:** same as classical soundness but adversary is quantum

$$\forall t_{\text{ARG}}\text{-time } \mathbf{QUANTUM} \text{ adversary } \tilde{P}, \Pr \left[ \langle \tilde{P}, V \rangle = 1 \right] \leq \epsilon_{\text{ARG}}(t_{\text{ARG}})$$

# On quantum rewinding



For many years: can rewind  $O(1)$  times [Wat06, Unr12, Unr16b]

**Problem: Kilian's protocol needs many rewinds**

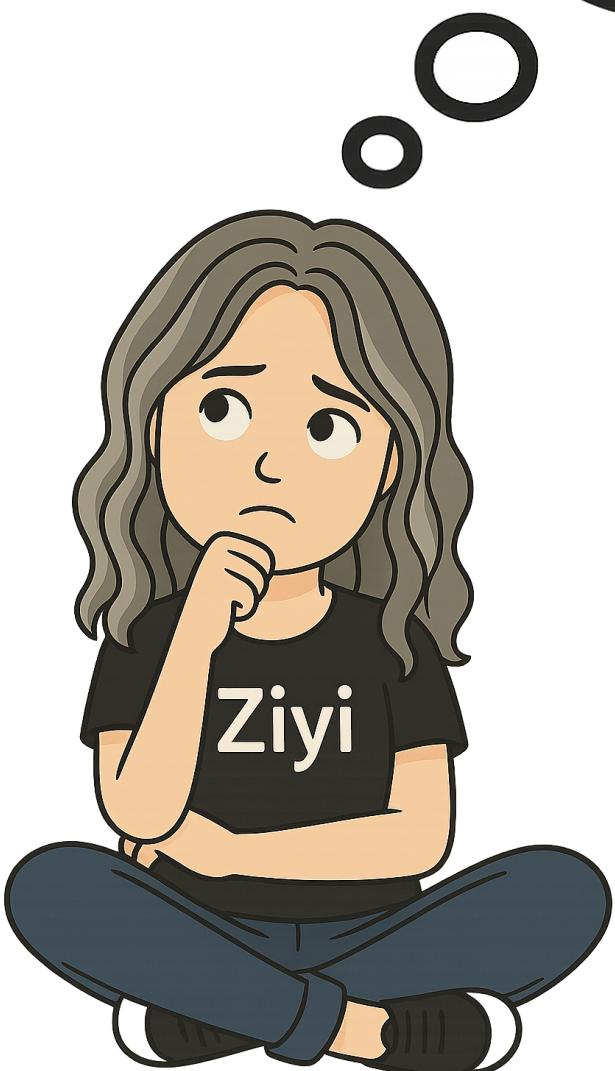
Recent new tools for quantum rewinding [CMSZ21]:  
“repair” the state instead of “rewind”

⇒ post-quantum security of Kilian's protocol

21

Adapting for IBCS protocol runs into challenges

But rewinding  
is everywhere in  
crypto, how did  
people prove  
anything without it?



# Post-quantum security of IBCS protocol



Technical contribution: We build on [CMSZ21] and more...

**Theorem.**  $\forall \epsilon > 0$ ,

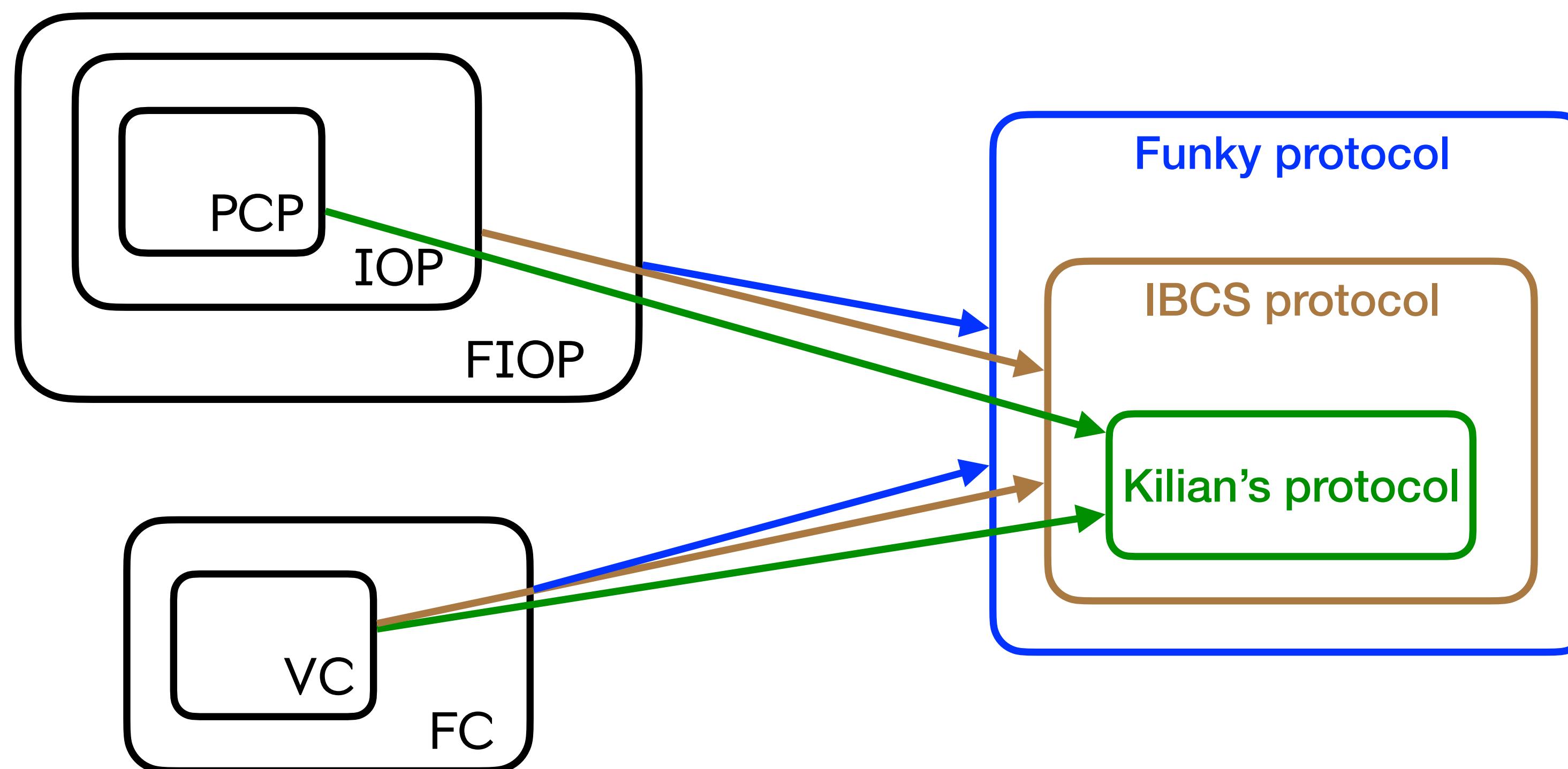
$$\epsilon_{\text{ARG}}^{\text{PQ}}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + k \cdot l \cdot \epsilon_{\text{VCCollapse}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = \text{poly}(t_{\text{ARG}} \cdot l/\epsilon).$$

Extra  $l$  factor: cost of quantum rewinding

$$\text{IBCS soundness: } \epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + k \cdot \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot l/\epsilon).$$

**Corollary:** post-quantum secure succinct arguments in the standard model (no oracles), with the best asymptotic complexity known.

# Funky protocol: Construction from all probabilistic proofs

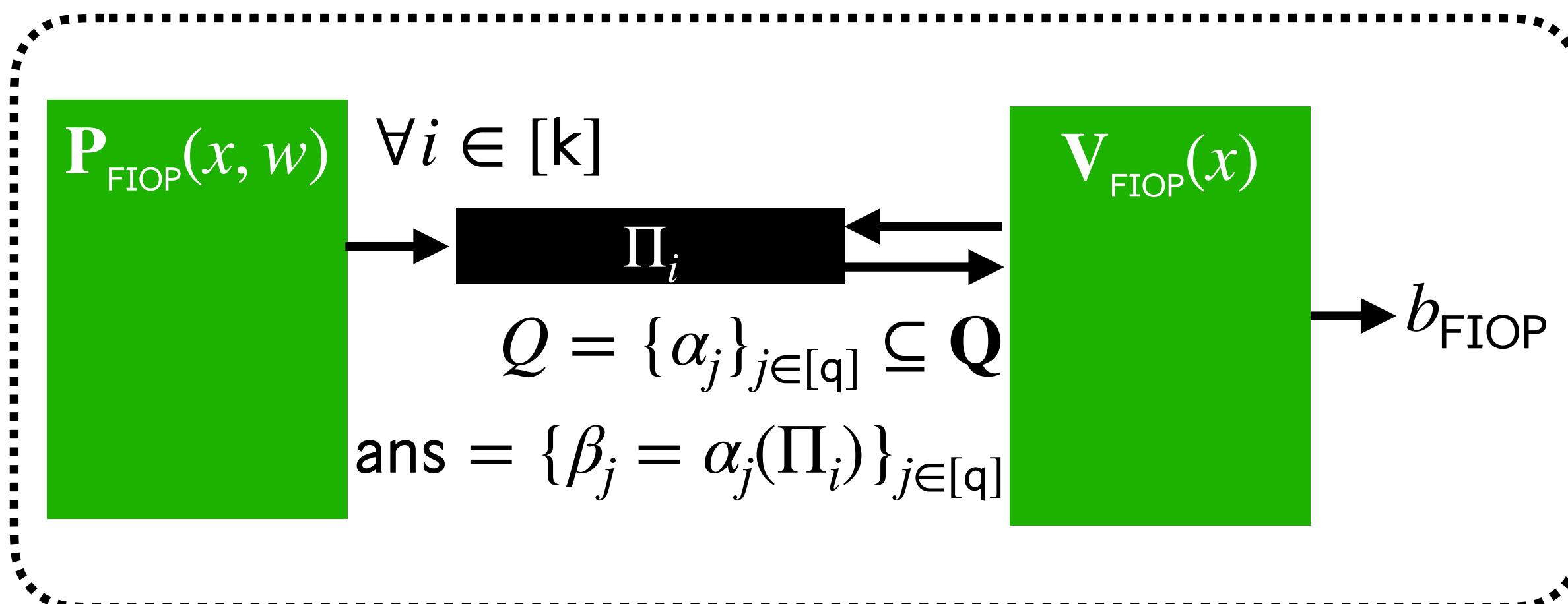


# Building blocks

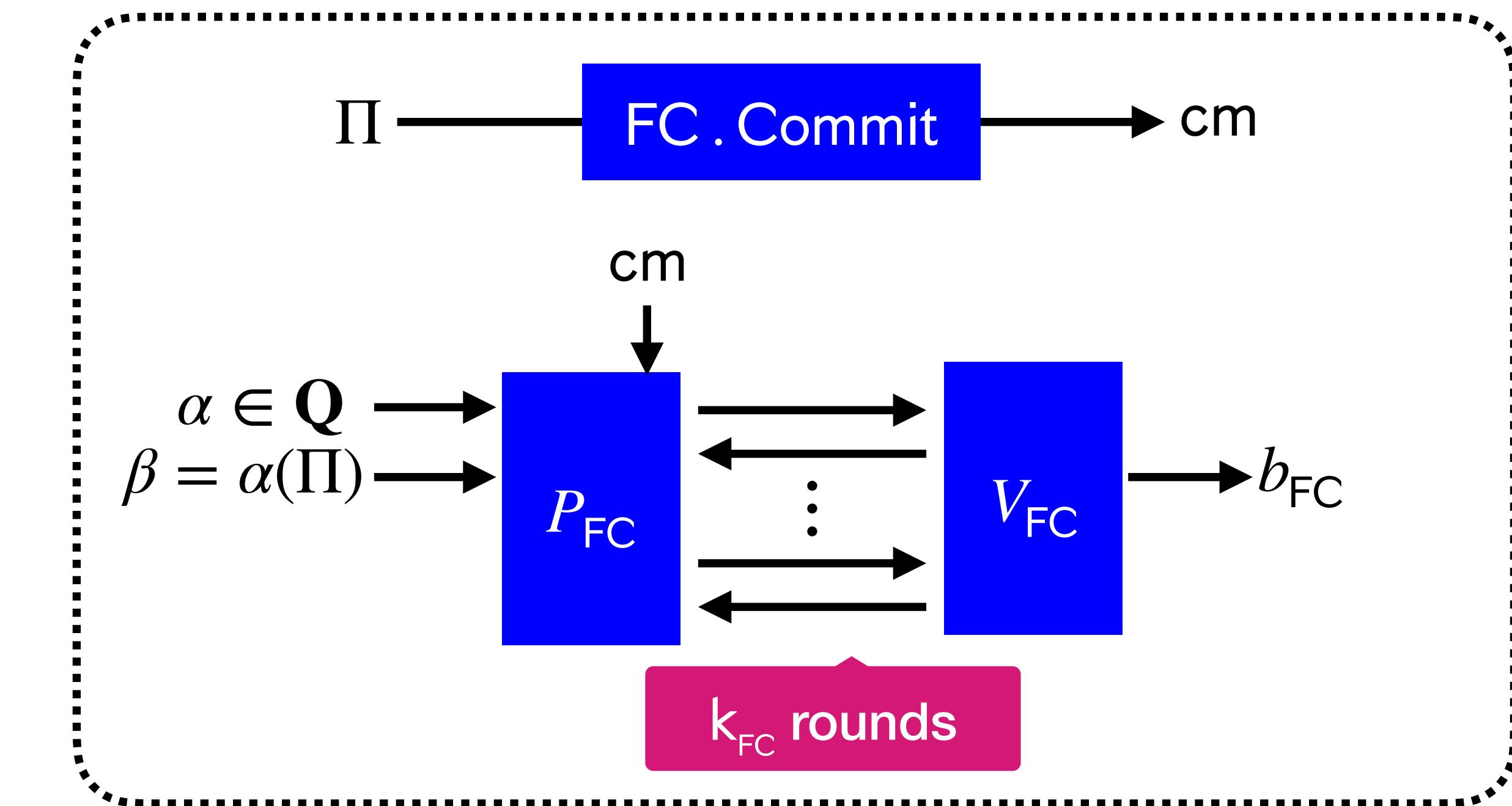
**Building block #1:** query class  $\mathbf{Q}$

$$- \mathbf{Q} \subseteq \{\alpha: \Sigma^\ell \rightarrow \mathbb{D}\}$$

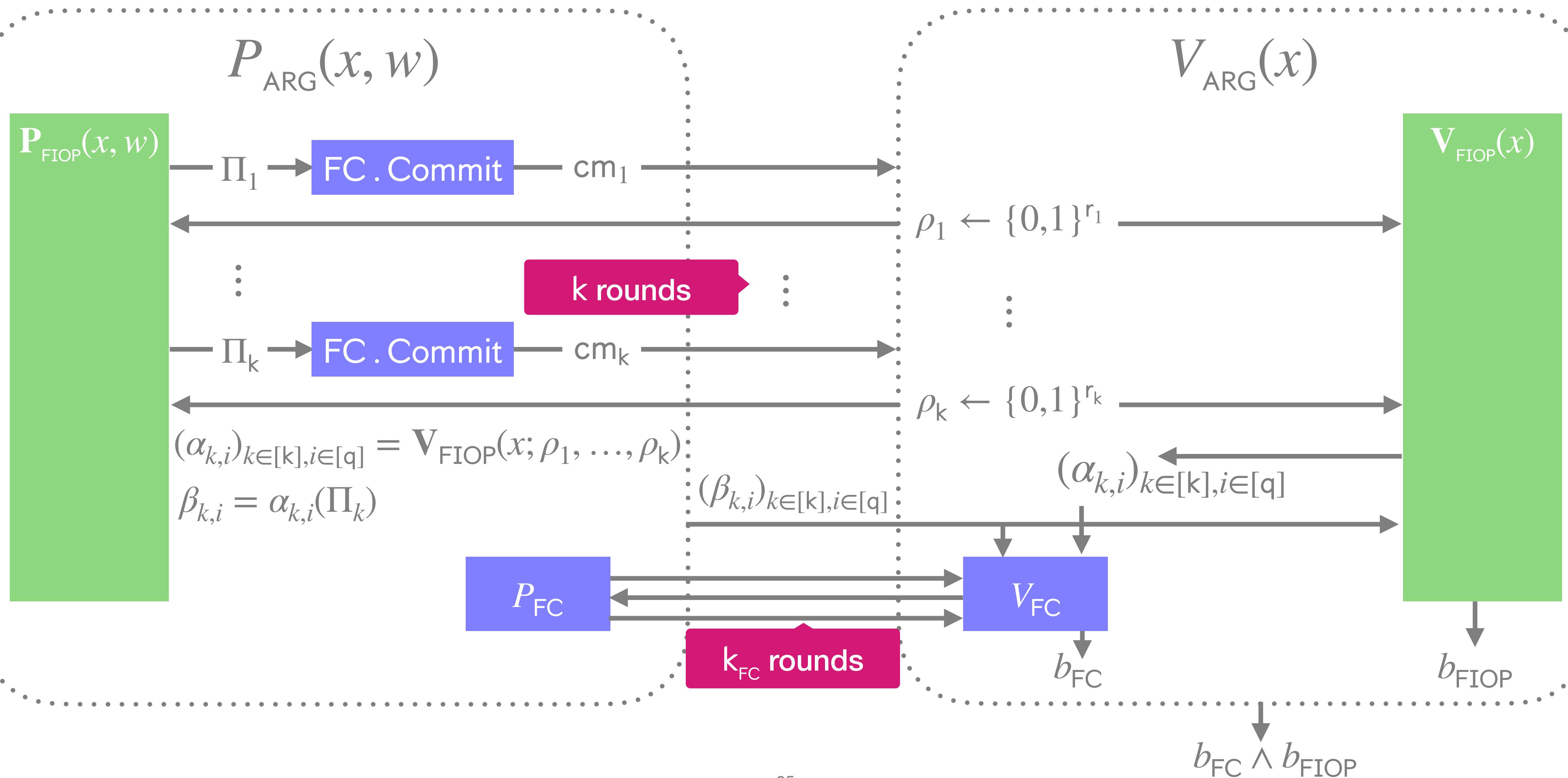
**Building block #2:** functional interactive oracle proof (FIOp)



**Building block #3:** functional commitment scheme (FC)



# Funky protocol



# Special cases of the Funky protocol

	Proof string	Query class	Answer
<b>PCP+VC</b> [Kilian92] <b>IOP+VC</b> [BCS16,CDGS23]	$\Pi \in \Sigma^\ell$	point queries $\mathbf{Q}_{\text{point}}$	$\beta = \Pi[\alpha] \text{ for } \alpha \in [\ell]$
<b>LPCP+LC</b> [LM19]	$\Pi \in \mathbb{F}^\ell$	linear queries $\mathbf{Q}_{\text{lin}}$	$\beta = \sum_{i \in [\ell]} \Pi[i] \cdot \alpha[i] \text{ for } \alpha \in \mathbb{F}^\ell$
<b>PIOP+PC</b> [CHM+20,BFS20]	$\Pi \in \mathbb{F}[X]^{\leq D}$	evaluation queries on polynomials $\mathbf{Q}_{\text{poly}}$	$\beta = \sum_{i \in [\ell]} \Pi[i] \cdot \alpha^{i-1} \text{ for } \alpha \in \mathbb{F}$
<b>PIOP*+PC*</b> [GWC19]	$\Pi \in (\mathbb{F}[X]^{\leq D})^{m+n}$ $= (f_1, \dots, f_m, g_1, \dots, g_n)$	evaluation queries on structured polys $\mathbf{Q}_{\text{poly}*}$	$\beta = \sum_{k \in [n]} h_k(f_1(\alpha), \dots, f_m(\alpha)) \cdot g_k(\alpha)$

Beyond Funky: Bulletproofs (and other sumcheck-based arguments), linear-only encodings [BCIOP13, GGPR13, Groth16], ...

# Special cases of the Funky protocol

Proof string	Query class	Answer
Funky protocol is everywhere		
 <b>Succinct</b>	 RISC ZERO	 <b>Ligero</b>
 <b>STARKWARE</b>	 <b>Aztec</b>	 <b>VALIDA</b>
 <b>polygon</b>	 <b>NEXUS</b>	<b>J[irreducible</b> ... $k(\alpha)$

Beyond Funky: Bulletproofs (and other sumcheck-based arguments), linear-only encodings [BCIOP13, GGPR13, Groth16], ...

# Which security property for FC?

Earlier in this talk **IOP+VC**

$$\epsilon_{\text{ARG}} \approx \epsilon_{\text{IOP}} + \epsilon_{\text{VC}}^{\text{PB}}$$

**Vector Commitments**

position binding:  $\Pr \left[ \begin{array}{l} \beta_1 \neq \beta_2 \\ \wedge \forall i : \text{FC} . \text{Check}(\text{pp}, \text{cm}, \alpha_i, \beta_i, \text{pf}_i) = 1 \end{array} \mid (\text{cm}, \alpha, \beta_1, \text{pf}_1, \beta_2, \text{pf}_2) \leftarrow A(\text{pp}) \right] \leq \epsilon$

[LM19] **LPCP+LC**

$$\epsilon_{\text{ARG}} \approx \epsilon_{\text{LPCP}} + \epsilon_{\text{LC}}^{\text{FB}}$$

**Linear Commitments**

function binding:  $\Pr \left[ \begin{array}{l} \nexists \Pi : \forall i : \langle \alpha_i, \Pi \rangle = \beta_i \\ \wedge \forall i : \text{FC} . \text{Check}(\text{pp}, \text{cm}, \alpha_i, \beta_i, \text{pf}_i) = 1 \end{array} \mid (\text{cm}, (\alpha_i, \beta_i, \text{pf}_i)_{i \in [n]}) \leftarrow A(\text{pp}) \right] \leq \epsilon$

[CHMMW20, BFS20] **PIOP+PC**

$$\epsilon_{\text{ARG}} \approx \epsilon_{\text{PIOP}} + \kappa_{\text{PC}}$$

**Polynomial Commitments**

binding? strong correctness? interpolation binding? extractability?

[KZG10]

[AJMMS23]

[CHM+20, BFS20]

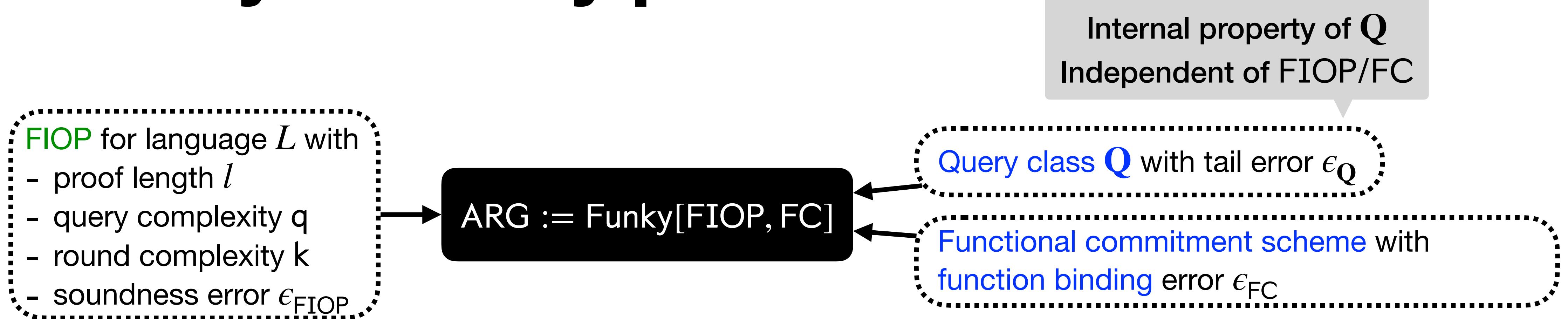
Too strong

**Functional Commitments**

function binding:

$\Pr \left[ \begin{array}{l} \nexists \Pi : \forall i : \alpha_i(\Pi) = \beta_i \\ \wedge \forall i : \text{FC} . \text{Check}(\text{pp}, \text{cm}, \alpha_i, \beta_i, \text{pf}_i) = 1 \end{array} \mid (\text{cm}, (\alpha_i, \beta_i, \text{pf}_i)_{i \in [n]}) \leftarrow A(\text{pp}) \right] \leq \epsilon$

# Security of Funky protocol



**Theorem.**  $\forall N \in \mathbb{N}$ ,

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{FIOP}} + k \cdot \epsilon_{\text{FC}}(t_{\text{FC}}) + k \cdot \epsilon_Q(l, N), \text{ where } t_{\text{FC}} = O(t_{\text{ARG}} \cdot N).$$

TLDR:

- A “tight” security notion for FC schemes
- Concrete and tight bounds using tail errors

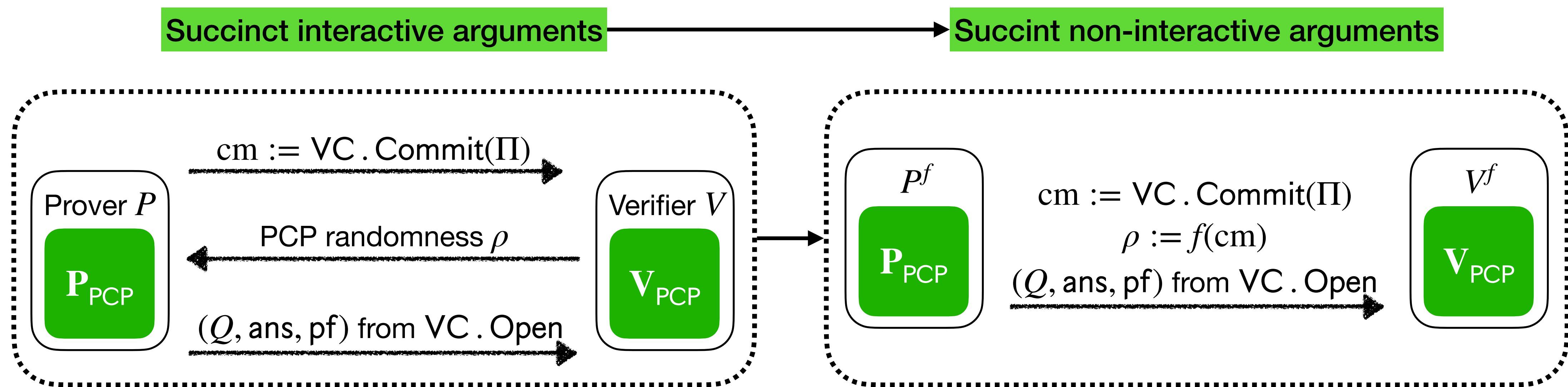
$\epsilon_{Q_{\text{point}}}(l, N) = l/N \implies$  recovers the bounds for Kilian’s protocol and IBCS protocol

# Fiat-Shamir security: From succinct arguments to SNARGs

# Fiat-Shamir transformation

Random oracle:  $\mathcal{O} = \{\mathcal{O}_\lambda\}_{\lambda \in \mathbb{N}}$

$\mathcal{O}_\lambda$ : uniform distribution over  $\{f: \{0,1\}^* \rightarrow \{0,1\}^\lambda\}$



Central question: Is security preserved after the Fiat-Shamir transformation?

In general no [CY24]:  $\epsilon_{\text{NARG}}(x, t, m) \leq (m+1)^k \cdot \epsilon_{\text{ARG}}(x, t)$

RO queries

$k$  might be superconstant!

# Fiat-Shamir security



**Theorem.**  $\forall N \in \mathbb{N}$ ,

$$\epsilon_{\text{NARG}}(t_{\text{ARG}}, m_{\text{ARG}}) \leq \epsilon_{\text{FIOP}}^{\text{FS}}(O(m_{\text{ARG}})) + k \cdot \epsilon_{\text{FC}}^{\text{FSFB}}(t_{\text{FC}}, m_{\text{FC}}) + k \cdot \epsilon_{\mathbf{Q}}(l, N), \text{ where } \begin{cases} t_{\text{FC}} = O(t_{\text{ARG}} \cdot N) \\ m_{\text{FC}} = O(m_{\text{ARG}} \cdot k \cdot N) \end{cases}.$$

A theorem that generalizes everything we saw (except post-quantum)

**Corollary:** security analysis of Plonk [GWC19] from falsifiable assumption



...

# Open problems

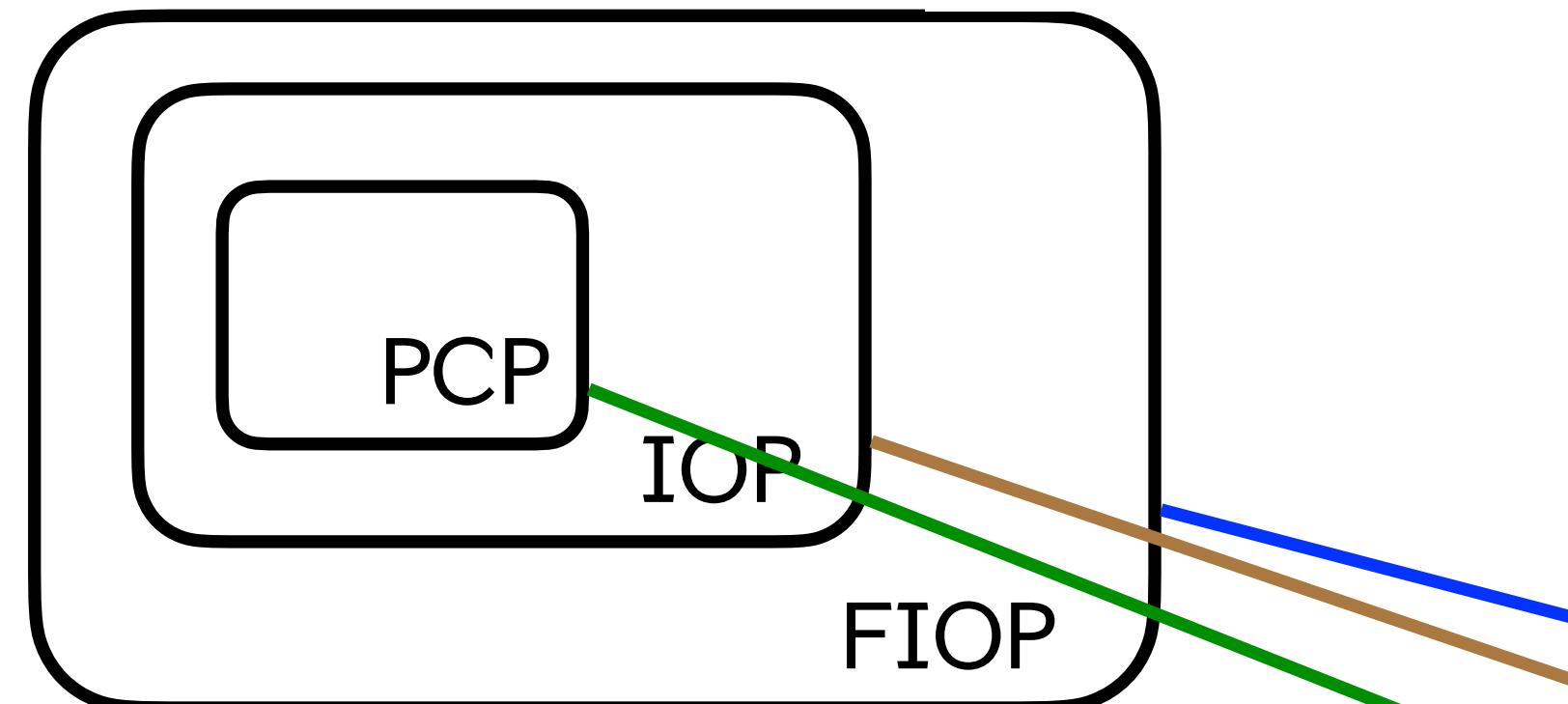
SNARGs for NP via Fiat–Shamir in the Plain Model

Ziyi Guan  
ziyi.guan@epfl.ch  
EPFL

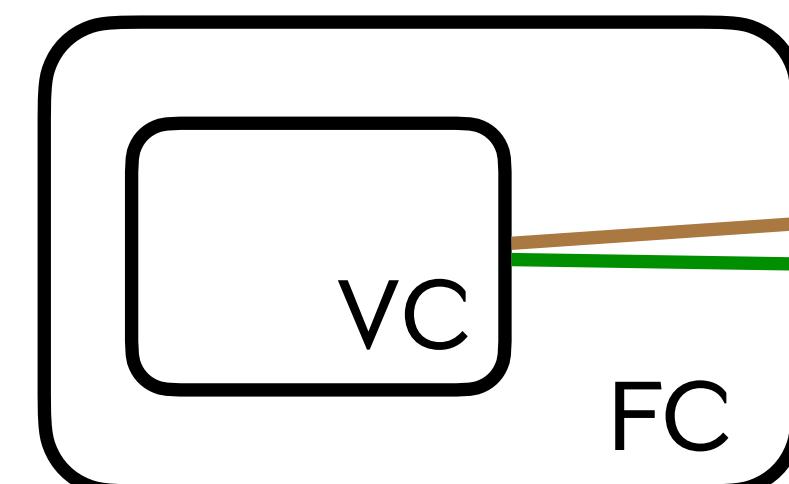
Eylon Yogev  
eylon.yogev@biu.ac.il  
Bar-Ilan University

Standard model Fiat–Shamir wo/ iO?

Probabilistic proofs



Commitment schemes



Funky protocol

- Soundness
- Fiat–Shamir soundness

Quantum analogue?

Post-quantum security?

Expected-time regime?

Practical security in idealized models?

IBCS protocol

- Soundness
- Private-coin IOPs
- Post-quantum soundness

Public-query IOPs?

Kilian's protocol

- Soundness
- Lower bounds on soundness

Kilian vs. Sigma protocols?



# References

[BG08]: Boaz Barak and Oded Goldreich. “Universal Arguments and their Applications”. CCC ’02.

[BL02]: Boaz Barak and Yehuda Lindell. “Strict polynomial-time in simulation and extraction”. STOC ’02.

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